SHAPE-OPTIMIZATION OF MULTI-MODAL RESONATORS ACCOUNTING FOR ROOM/RESONATOR ACOUSTICAL COUPLING

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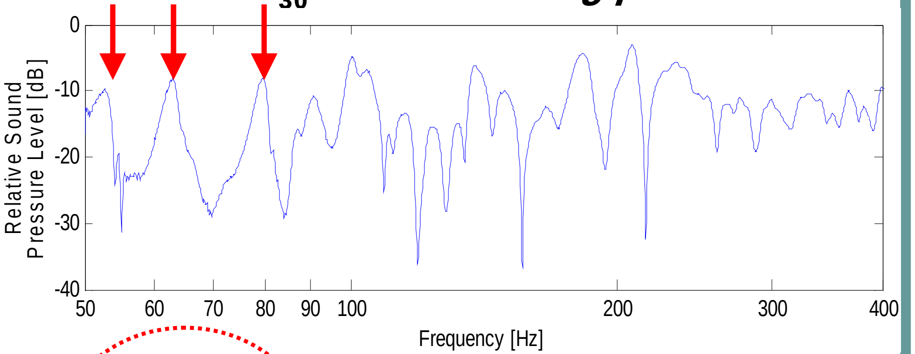
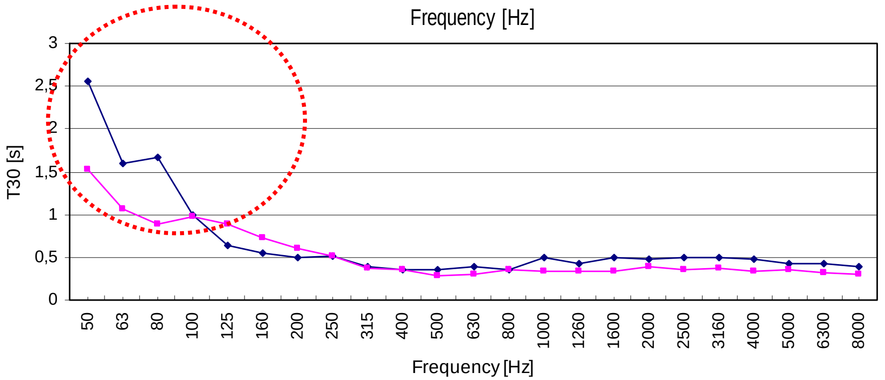
IN THE LOW-FREQUENCY RANGE, RESPONSIVE CONTROL ROOM RESONANCES SHOULD BE AVOIDED

Room acoustical response and T_{30} at the listening position





For the usual small volumes of control rooms, standing waves arise at <u>audible frequencies</u> well below the Schroeder frequency

Extended modal decay times
Non-uniformity of the frequency response

→ **Sound Coloration**

CURRENT TECHNIQUES FOR LOW-FREQUENCY DESIGN

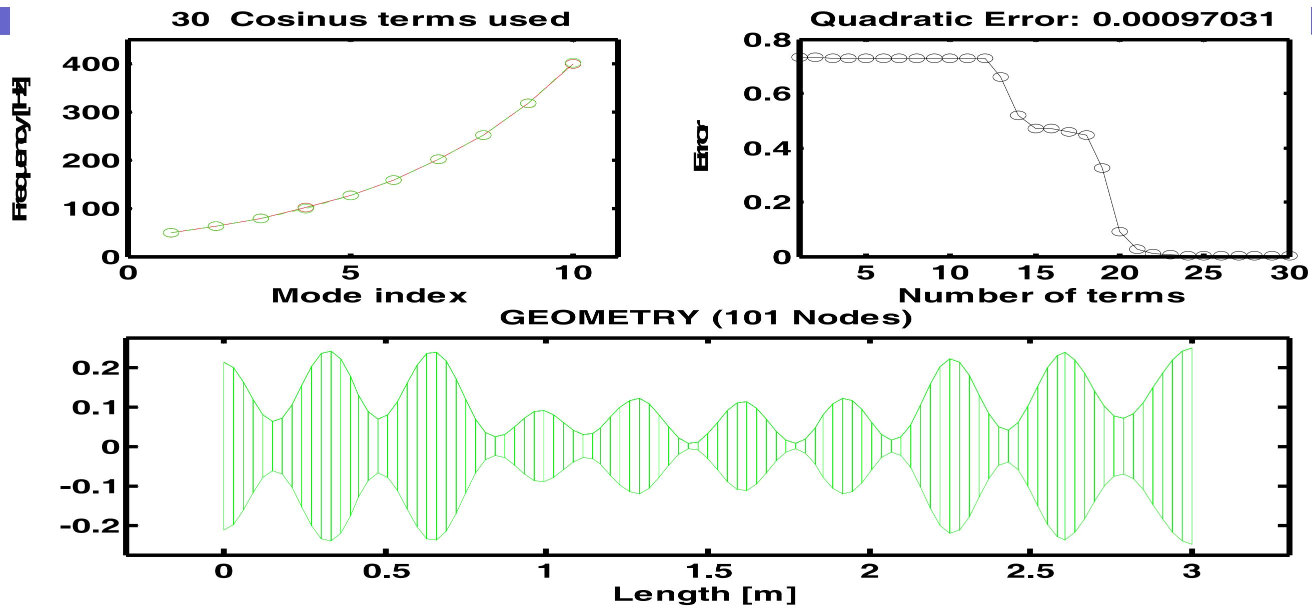
- **Choice of room shape and dimensions**
- **Choice of loudspeaker and listener locations**
- **Panel absorbers**
- **Helmholtz bass-trapping resonators**

WHY NOT USE OPTIMIZED MULTI-MODAL BASS-TRAPPING RESONATORS ?

Design of duct cross sectional areas in bass-trapping resonators for control rooms

Inácio, Henrique & Antunes
***Noise Control Engineering Journal* 55 (2007) 172-182.**

OPTIMAL SHAPES TO PRODUCE A GIVEN TARGET SET OF MODAL FREQUENCIES FOR THE ACOUSTICAL RESONATOR



However, modal frequencies are only part of the problem:

(Morse & Ingard, 1968 ; Fahy & Schofield, 1980)

Damping phenomena
Acoustical room & resonator modeshapes
Resonator locations

↓

Room / resonator(s) coupling efficiency

THE RESONATOR OPTIMIZATION PROBLEM SHOULD BE SOLVED FOR THE ROOM / RESONATOR(S) COUPLED SYSTEM

In 2007 (ISRA 2007 Sevilla) a sub-structure computational approach to the coupled problem was presented:

Antunes & Inácio (2007) - A Theoretical Analysis of Multi-Modal Bass-Trapping Resonators Coupled to Control-Room Acoustics.

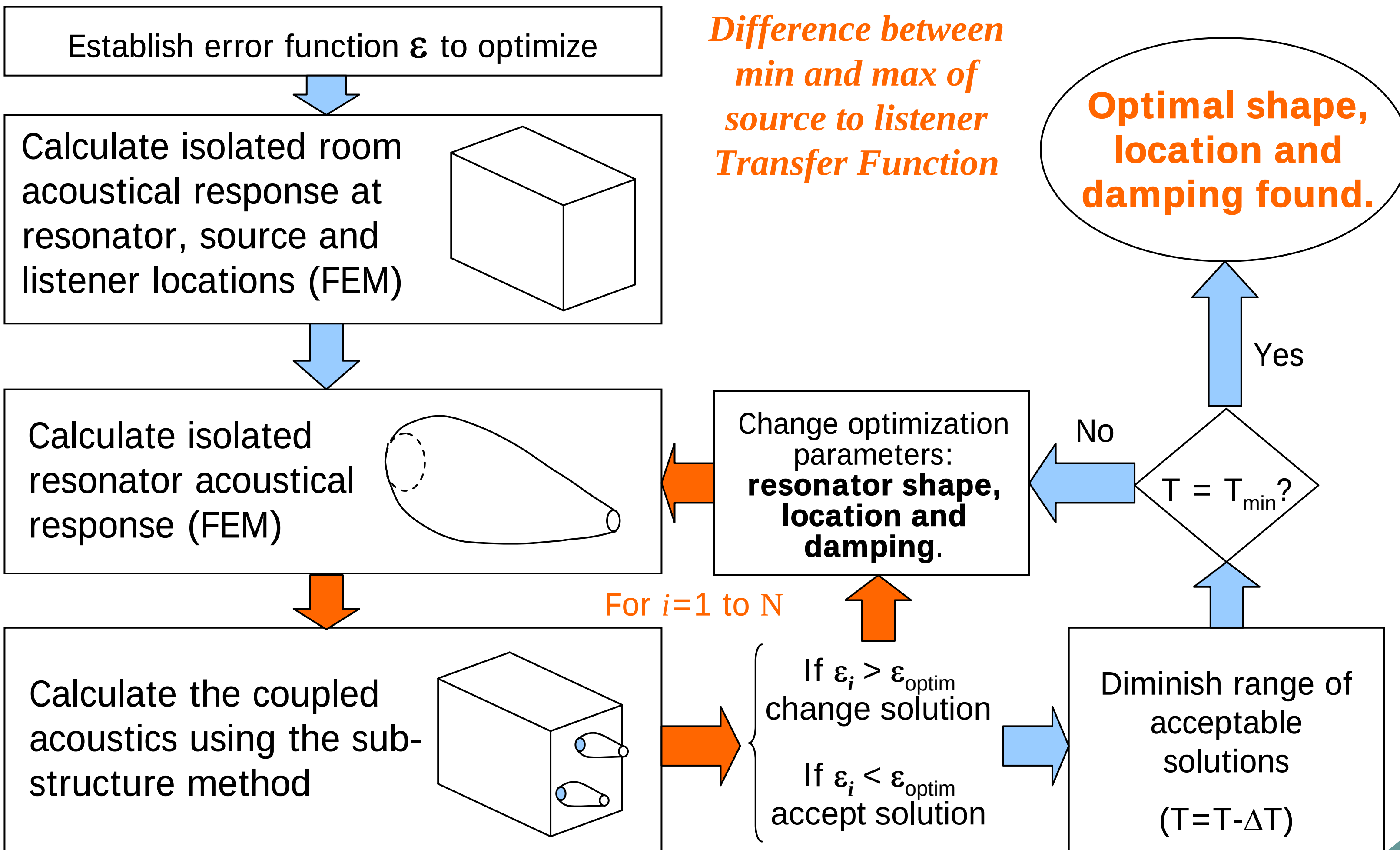
Computer-intensive methods (FEM, BEM) , with thousands of DOFs, are not ideally suited for the coupled room/resonator computations needed during the optimization procedure.

Sub-structure / component-mode-synthesis techniques are much more economical, but they have been used more for structural than acoustical problems.

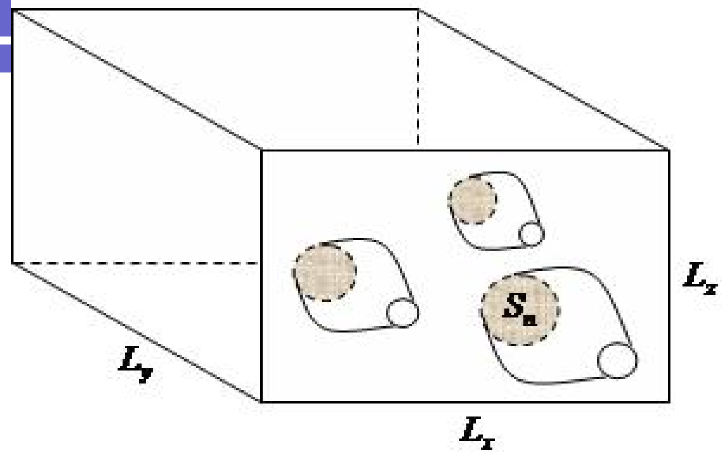
Furthermore, if the **modal basis** are well chosen, only the sub-system modes of **component(s) to be shape-optimized** are recomputed at each optimization iteration, while those of the room are computed only once.

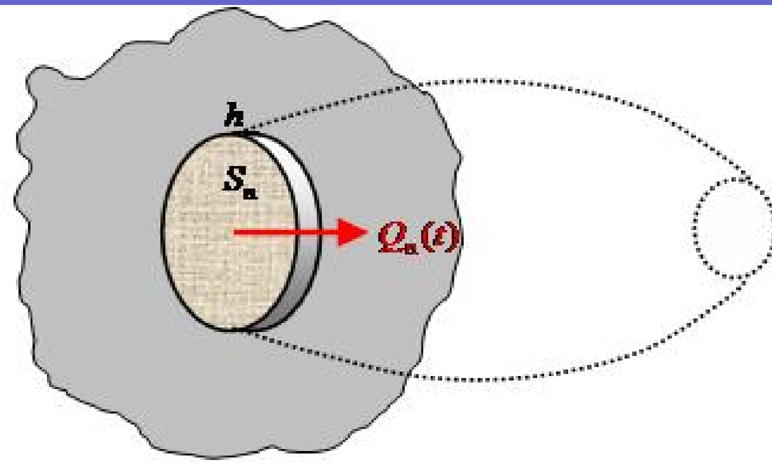
OPTIMIZATION PROCEDURE

Global optimization method used: Simulated Annealing



CONSERVATIVE MODEL FOR COUPLED ROOM / RESONATORS





Room pressure field

$$\ddot{p}_r(\vec{s}_r,t) - c_0^2\nabla^2 p_r(\vec{s}_r,t) = c_0^2\rho_0\left[\sum_{n=1}^{N} S_n\ddot{\xi}_n(t)\,\delta(\vec{s}_r-\vec{s}_r^{\,n}) + \dot{Q}_e(t)\,\delta(\vec{s}_r-\vec{s}_r^{\,e})\right]$$

Resonator(s) pressure field

$$\ddot{p}_n(\vec{s}_n,t) - c_0^2\nabla^2 p_n(\vec{s}_n,t) = -c_0^2\rho_0 S_n\ddot{\xi}_n(t)\,\delta(\vec{s}_n-\vec{s}_n^{\,r}) \quad ; \quad n=1,2,\ldots,N$$

Momentum balance at the interface(s)

$$\rho_0 h S_n\,\ddot{\xi}_n(t) = S_n\left[p_n(\vec{s}_n^{\,r},t) - p_r(\vec{s}_r^{\,n},t)\right] \quad ; \quad n=1,2,\ldots,N$$

EQUIVALENT PENALTY FORMULATION

$$\ddot{\xi}_n(t) = \frac{1}{\rho_0 h}\left[p_n(\vec{s}_n^{\,r},t) - p_r(\vec{s}_r^{\,n},t)\right] \quad ; \quad n = 1,2,\ldots,N$$

$$\ddot{p}_r(\vec{s}_r,t) - c_0^2\nabla^2 p_r(\vec{s}_r,t) = \frac{c_0^2}{h}\sum_{n=1}^{N} S_n\left[p_n(\vec{s}_n,t)\,\delta(\vec{s}_n - \vec{s}_n^{\,r}) - p_r(\vec{s}_r,t)\,\delta(\vec{s}_r - \vec{s}_r^{\,n})\right]$$

$$\ddot{p}_n(\vec{s}_n,t) - c_0^2\nabla^2 p_n(\vec{s}_n,t) = -\frac{c_0^2}{h} S_n\left[p_n(\vec{s}_n,t)\,\delta(\vec{s}_n - \vec{s}_n^{\,r}) - p_r(\vec{s}_r,t)\,\delta(\vec{s}_r - \vec{s}_r^{\,n})\right]$$

$n = 1,2,\ldots,N$

Penalty parameter: $\frac{c_0^2}{h}$

MODAL FORMULATION

The sub-system modal basis are those of the room and resonator(s) closed at the interfaces

$$p_r(\vec{s}_r,t)=\sum_{m=1}^{M_r}\phi_m^{(r)}(\vec{s}_r)P_m^{(r)}(t) \quad \text{and} \quad p_n(\vec{s}_n,t)=\sum_{m=1}^{M_n}\phi_m^{(n)}(\vec{s}_n)P_m^{(n)}(t) \quad ; \quad n=1,2,\dots,N$$

⇓

$$A_k^{(r)}\ddot{P}_k^{(r)}(t)+B_k^{(r)}P_k^{(r)}(t)=c_0^2\rho_0\left[\sum_{n=1}^{N}S_n\,\ddot{\xi}_n(t)\,\phi_k^{(r)}(\vec{s}_r^{\,n})+\dot{Q}_e(t)\,\phi_k^{(r)}(\vec{s}_r^{\,e})\right] \quad ; \quad k=1,2,\dots,M_r$$

$$A_k^{(n)}\ddot{P}_k^{(n)}(t)+B_k^{(n)}P_k^{(n)}(t)=-c_0^2\rho_0 S_n\,\ddot{\xi}_n(t)\,\phi_k^{(n)}(\vec{s}_n^{\,r}) \quad ; \quad k=1,2,\dots,M_n \quad ; \quad n=1,2,\dots,N$$

$$\ddot{\xi}_n(t)=\frac{1}{\rho_0 h}\left[\left(\sum_{m=1}^{M_n}\phi_m^{(n)}(\vec{s}_n^{\,r})P_m^{(n)}(t)\right)-\left(\sum_{m=1}^{M_r}\phi_m^{(r)}(\vec{s}_r^{\,n})P_m^{(r)}(t)\right)\right] \quad ; \quad n=1,2,\dots,N$$

DISSIPATIVE PROBLEM

(a) Damping coefficients $\zeta_k^{(r)}$ and $\zeta_k^{(n)}$ in the modal equations :

$$A_k^{(r)}\ddot{P}_k^{(r)}(t)+Z_k^{(r)}\dot{P}_k^{(r)}(t)+B_k^{(r)}P_k^{(r)}(t)=c_0^2\rho_0\left[\sum_{n=1}^{N}S_n\,\ddot{\xi}_n(t)\,\phi_k^{(r)}(\vec{s}_r^{\,n})+\dot{Q}_e(t)\,\phi_k^{(r)}(\vec{s}_r^{\,e})\right]\quad;\quad k=1,2,\ldots,M_r$$

$$A_k^{(n)}\ddot{P}_k^{(n)}(t)+Z_k^{(n)}\dot{P}_k^{(n)}(t)+B_k^{(n)}P_k^{(n)}(t)=-c_0^2\rho_0 S_n\,\ddot{\xi}_n(t)\,\phi_k^{(n)}(\vec{s}_n^{\,r})\quad;\quad k=1,2,\ldots,M_n\quad;\quad n=1,2,\ldots,N$$

(b) At the room / resonator interface(s) (viscous phenomena, use of damping porous materials) with "acoustic resistance" *R*.

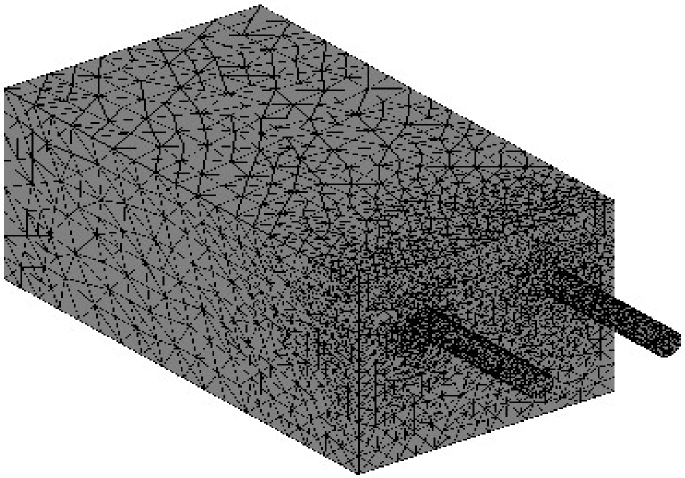
$$\rho_0 h S_n\,\ddot{\xi}_n(t)+R S_n\dot{\xi}_n(t)=S_n\left[p_n(\vec{s}_n^{\,r},t)-p_r(\vec{s}_r^{\,n},t)\right]\quad;\quad n=1,2,\ldots,N$$

⇩

LEADS TO AN EIGENVALUE PROBLEM

$$[\,A\,]\{\ddot{V}(t)\}+[\,D\,]\{\dot{V}(t)\}+[\,B\,]\{V(t)\}=\{E\}\,c_0^2\rho_0\dot{Q}_e(t)$$

A SIMPLE EXAMPLE OF THE SUB-STRUCTURE METHOD
"SHOE-BOX" ROOM & 2 CYLNDRICAL RESONATORS



$$\begin{cases} L_x = 5\text{ m}, L_y = 9\text{ m}, L_z = 4\text{ m} \\ L = 3 \,\&\, 4.5\text{ m}, D = 0.5 \,\&\, 1\text{ m} \end{cases}$$

Room modal basis:

$$\begin{cases} f_{ijk}^{(r)} = \dfrac{c_0}{2}\left[\left(\dfrac{i}{L_x}\right)^2 + \left(\dfrac{j}{L_y}\right)^2 + \left(\dfrac{k}{L_z}\right)^2\right]^{1/2} \\ \phi_{ijk}^{(r)}(x,y,z) = \cos\dfrac{i\pi x}{L_x}\cos\dfrac{j\pi y}{L_y}\cos\dfrac{k\pi z}{L_z} \end{cases} \quad ; \ (i,j,k = 0,1,2,\ldots)$$

Resonator modal basis:

$$\begin{cases} f_m^{(r)} = \dfrac{c_0 m}{2L} \\ \phi_m^{(n)}(s) = \cos\dfrac{m\pi s}{L} \end{cases} \quad ; \quad (m = 0,1,2,\ldots)$$

COMPARISON BETWEEN PRESENT METHOD AND FEM COMPUTATIONS

Modal frequencies of the isolated closed sub-systems (L=3 m , D=0.5 m)

Mode	1	2	3	4	5	6	7	8	9	10	11	12	13
Room	0.0	19.06	34.30	38.11	39.24	42.88	46.92	51.27	54.91	57.17	57.36	58.12	66.67
Resonators	0.0	57.17	114.3										

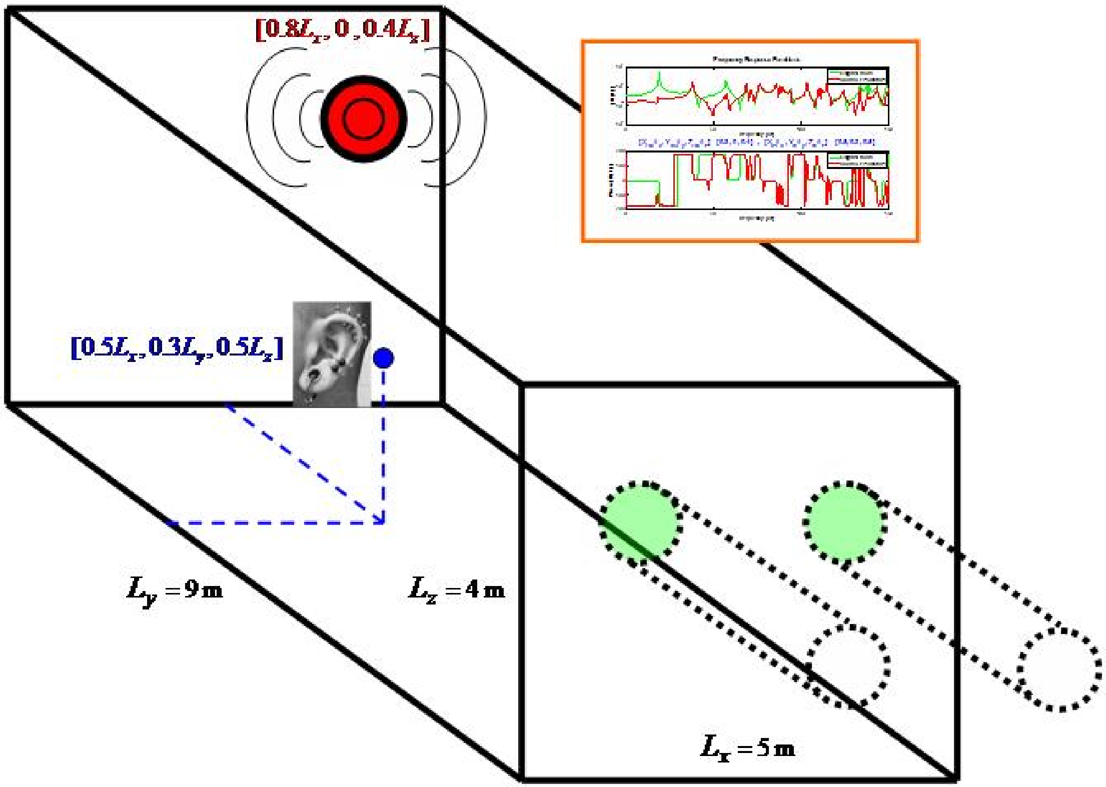
Modal frequencies of the coupled system (L=3 m , D=0.5 m)

About 200 modes used

Mode	1	2	3	4	5	6	7	8	9	10
Present approach	0.0	18.87	29.26	29.72	34.50	38.34	39.44	42.92	46.97	51.31
FEM	0.0	18.83	26.62	27.07	34.43	38.28	39.39	42.92	46.97	51.31

- **A few hundred modes against 10^5 FEM dofs**
- **Good precision results using 300 modes**
- **Three orders of magnitude faster than FEM**

FORCED RESPONSES TO A VOLUME VELOCITY SOURCE

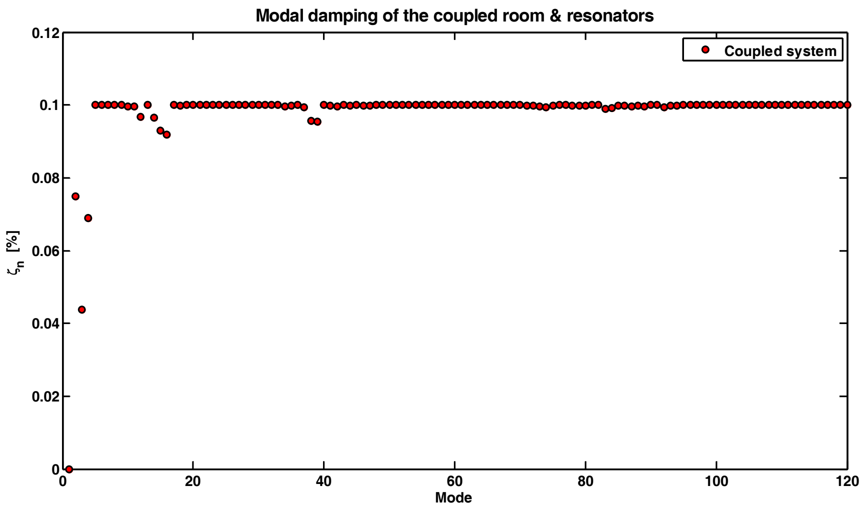


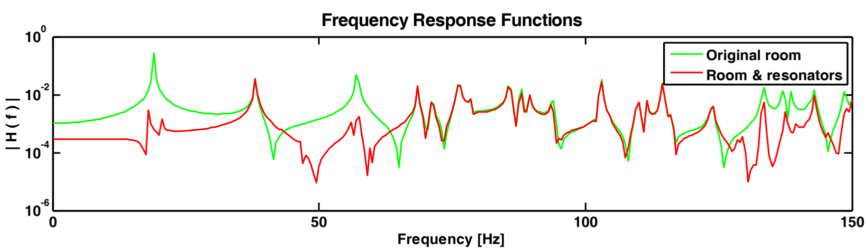
$$\{V(\omega)\} = \left([\,B\,] + i\omega [\,D\,] - \omega^2 [\,A\,] \right)^{-1} \{E\}\, i\omega c_0^2 \rho_0 Q_e(\omega)$$

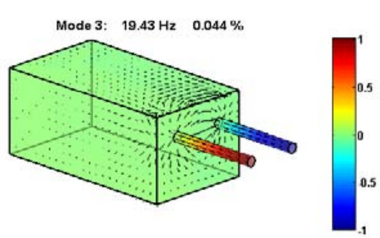
EFFECT OF THE ROOM / RESONATOR(S) INTERFACE DAMPING

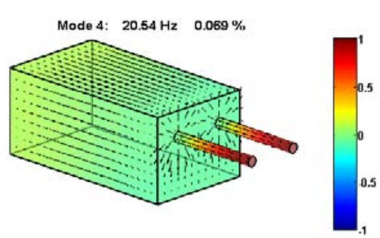
$L = 4.5\ \text{m}$ $D = 0.5\ \text{m}$ $\zeta_{\text{Room}} = \zeta_{\text{Reso}} = 0.1\%$

$R = 0$

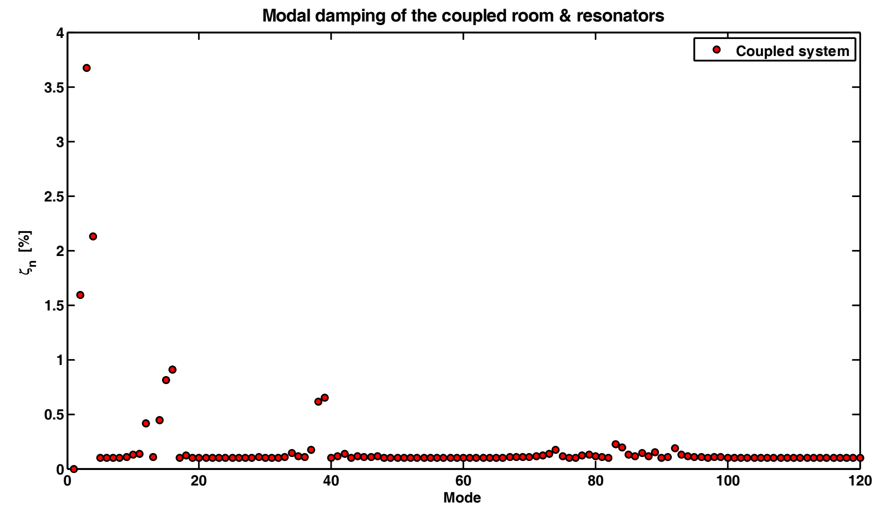


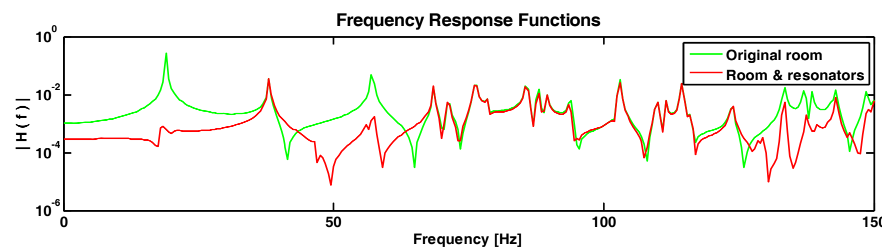


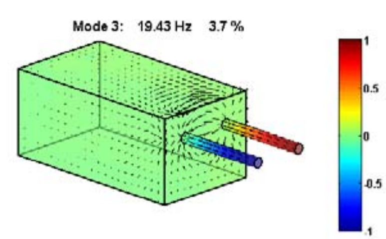


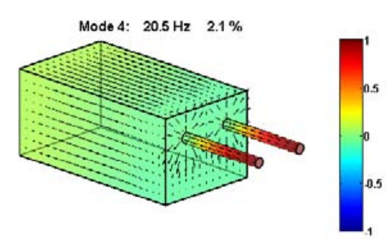


$R = 25\ \text{Ns/m}^3$









OPTIMIZATION PROCEDURE CALCULATIONS

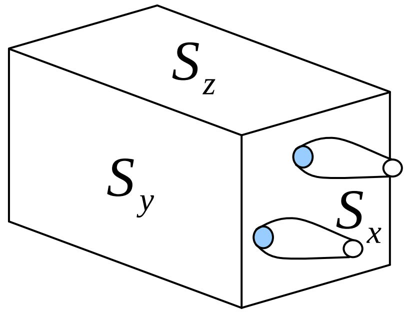
$L_x = 5\text{ m}; L_y = 9\text{ m}; L_z = 4\text{ m}$ $\zeta_{\text{Room}} = 0.5\%;$ $\zeta_{\text{Reso}} = 2.5\%$

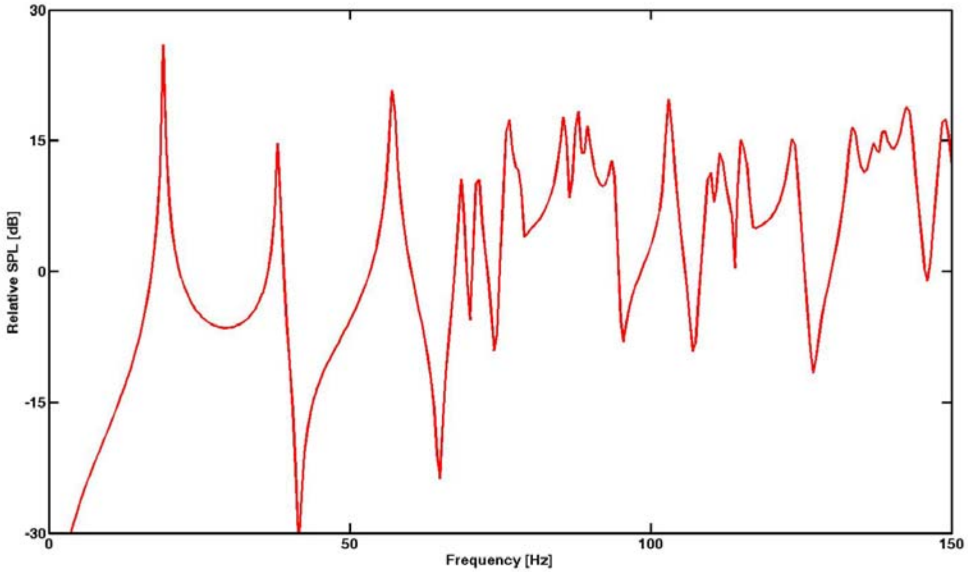
Objective:

Using two resonators applied at the S_x surface

find the shape, location and interface resistivity to minimize

Max(TF) – Min (TF) between 15 Hz to 150 Hz.





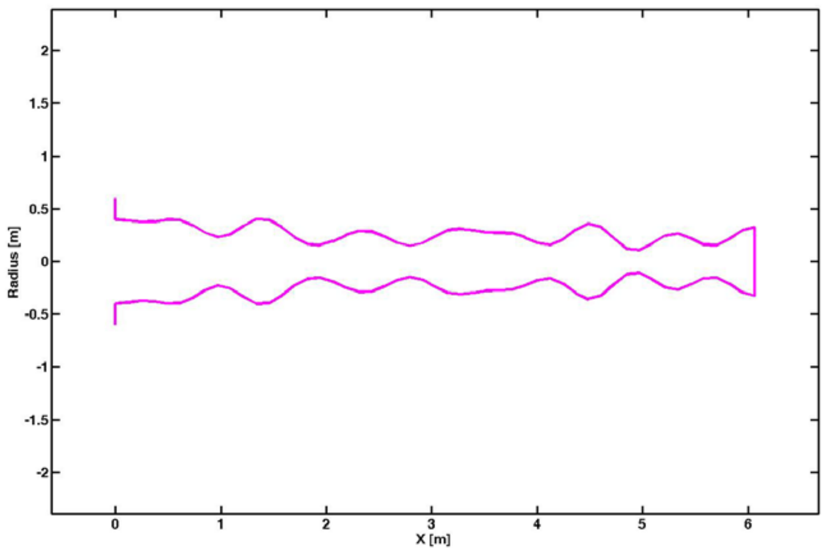
OPTIMIZATION PROCEDURE CALCULATIONS

$L_x = 5\text{ m}; L_y = 9\text{ m}; L_z = 4\text{ m}$ $\zeta_{\text{Room}} = 0.5\%;$ $\zeta_{\text{Reso}} = 2.5\%$

Optimal resonator shapes found

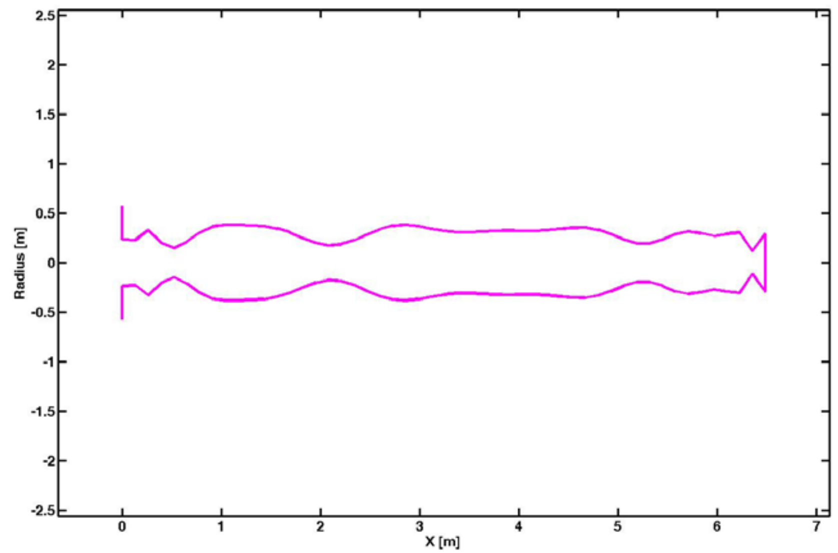
Cosine functions

$L_{res} = 6.0627$, $X_{res} = 0.69189$, $Y_{res} = 9$, $Z_{res} = 2.8246$, $R_{int} = 10000$



Chebyshev polynomials

$L_{res} = 6.4859$, $X_{res} = 1.1568$, $Y_{res} = 9$, $Z_{res} = 2.4571$, $R_{int} = 10000$

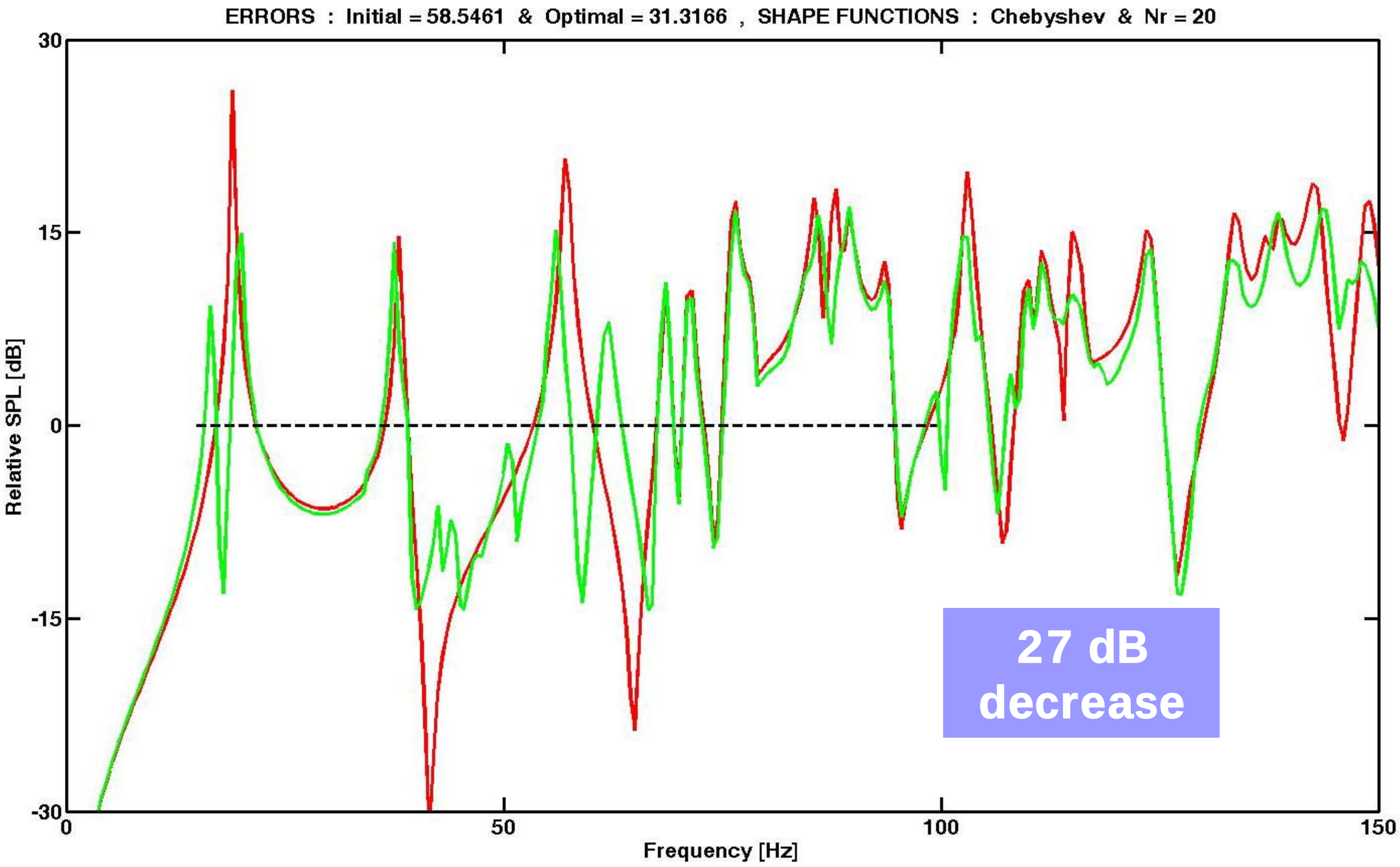


OPTIMIZATION PROCEDURE CALCULATIONS

$L_x = 5$ m; $L_y = 9$ m; $L_z = 4$ m $\zeta_{\text{Room}} = 0.5\%;$ $\zeta_{\text{Reso}} = 2.5\%$

Optimal transfer functions found

Chebyshev polynomials

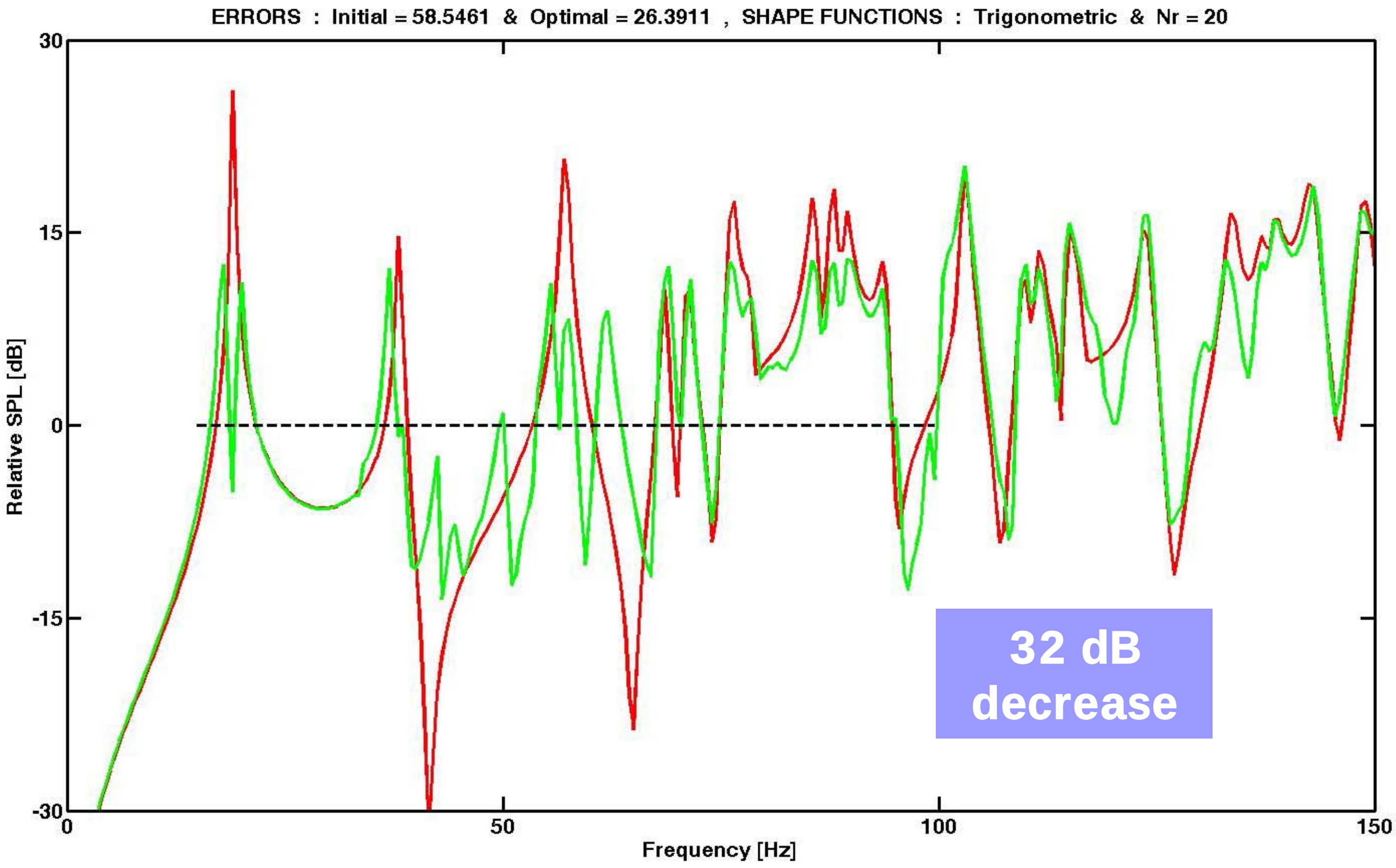


OPTIMIZATION PROCEDURE CALCULATIONS

$L_x = 5$ m; $L_y = 9$ m; $L_z = 4$ m $\qquad \zeta_{Room} = 0.5\%; \qquad \zeta_{Reso} = 2.5\%$

Optimal transfer functions found

Cosine functions



CONCLUSIONS

1) We have addressed the problem of optimizing the shape, locations and interface damping of bass-trapping resonators coupled to the acoustical response of a room.

2) The acoustical component mode synthesis method (developed in previous work) was implemented with a Simulated Annealing global optimization procedure.

3) Different solutions were found using different geometrical function that determine the resonator shape.

4) Results show that using two optimized multi-mode resonators at one of the room surfaces, the difference between maximum and minimum of the source to listener transfer function can be reduced by 30 dB.