

## Optimized Bass-Trapping Resonators for Control Rooms: A Preliminary Study

Octávio Inácio<sup>a</sup>, Luís Henrique<sup>b</sup>, José Antunes<sup>c</sup>

<sup>a,b</sup> Musical Acoustics Laboratory, Escola Superior de Música e das Artes do Espectáculo do Instituto Politécnico do Porto, Rua da Alegria, 503, 4000-045 Porto, Portugal

<sup>c</sup> Applied Dynamics Laboratory, Instituto Tecnológico e Nuclear, 2686 Sacavém codex, Portugal

<sup>a</sup>Octaviolnacio@esmae-ipp.pt; <sup>b</sup>LuisHenrique@esmae-ipp.pt; <sup>c</sup>jantunes@itn.pt

**Abstract** All small rooms, such as the ones specifically designed for amplified music listening like control rooms in recording studios, face the well-known problem of low-frequency over-enhancement by acoustic modes. For decades, several methods and devices have been developed to tackle this problem, some more efficient than others. Either by the use of Helmholtz resonators, membrane panels or tube-traps, among others, bass control devices (resonators) have typically been focused on a central frequency of maximum sound absorption usually spread over a determined bandwidth, depending on the system damping. Although it is possible to control more than one acoustic mode with a single damped resonator by adding porous materials in its construction, this is usually accomplished at the cost of a less efficient absorption. The number of controlled acoustic modes depends on the central frequency chosen, on the modal density in that frequency range, and damping. In this paper we suggest that the efficiency of such resonators may be significantly improved if, instead of using basic Helmholtz or cylindrical tube devices, more complex shape-optimized resonators are used, in order to cope with a larger number of undesirable acoustic modes. We apply optimization techniques, recently developed in our previous work, in order to obtain the optimal shapes for devices that resonate at a design set of acoustic eigenvalues, within imposed physical and/or geometrical constraints. Finite element models were implemented and coupled with optimization techniques in order to achieve this goal. We illustrate the proposed approach with several examples of resonator shapes and different design sets of absorption frequencies.

### INTRODUCTION

The acoustical design of small rooms for high fidelity sound reproduction requires particular attention to the control low-frequency resonances. The unbalance between over-enhancement of sound at these modal frequencies and the absence of room response at anti-resonances originates a detrimental lack of uniformity of the room acoustic response. This effect is more

pronounced for the frequency range where modal density and modal bandwidth (or modal damping) are very low. Additionally, the room dimensions may be such that modes occur at near frequencies, not only maximizing the resonance effect but also creating large frequency separation between modal frequencies.

These and other related problems have been tackled, with more or less efficiency, by the use of Helmholtz resonators, membrane panels or tube-traps, among many others. These bass control devices have typically been focused on a central frequency of maximum sound absorption usually spread over a determined bandwidth, depending on the system damping. Although it is possible to control more than one acoustic mode with a single damped resonator by adding porous materials in its construction, this is usually accomplished at the cost of a less efficient absorption. The number of controlled acoustic modes depends on the central frequency chosen, on the modal density in that frequency range, and damping. The degree of attenuation of the resonance effect is dependent not only on the number of such devices used, but also on their location in the room, ideally close to pressure antinodes of the mode to control. Helmholtz resonators have been particularly used in many different applications where an accurate control of a single frequency is desired. These resonators have been thoroughly studied since the 19<sup>th</sup> century beginning with the work of Helmholtz [1]. More recently, several researchers became interested in the optimization of the design and physical behaviour of such systems [2], on the effect of basic geometry changing on the resonant frequency [3], and on the acoustical coupling between the resonator and the room [4,5], to mention a few.

In this paper we suggest that the efficiency of such resonators may be significantly improved if, instead of using basic Helmholtz or cylindrical tube devices, more complex shape-optimized resonators are used, in order to cope with a larger number of undesirable acoustic modes. We apply optimization techniques recently developed in previous work [6,7], in order to obtain the optimal shapes for such devices that resonate at a design set of acoustic eigenvalues, within imposed physical and/or geometrical constraints. Finite element models were implemented and coupled with optimization techniques in order to achieve this goal. We illustrate the proposed approach with several examples of resonator shapes and different design sets of absorption frequencies. For this preliminary analysis we will focus only on the modal behaviour of the resonator isolated from the room. However, the complete analysis of this problem has to consider the frequency shifts and room modeshape distortion arising from the acoustical coupling between the room and the resonator. This aspect will be addressed elsewhere. Additionally, viscous boundary layer absorption effects which account for the damping at the neck of the resonators were not addressed in this model.

## **EXPERIMENTAL ANALYSIS OF TWO CONTROL ROOMS**

In order to obtain realistic examples of problematic acoustical resonance effects, two different control rooms were experimentally analysed. These control rooms belong to the College of Music and Performing Arts of the Polytechnic Institute of Porto, and are aimed to support the work of students of the Production and Music Technologies Degree, as well as the development of professional work by the Institute Audio Services. Both rooms have received acoustical treatment for the medium and high frequency range but have considerable problems in the reverberation time below 200 Hz. Figure 1 represents the results of reverberation time measurements carried out in both rooms using the monitor loudspeakers located on the mixing table and a microphone at the listener or mixing position.

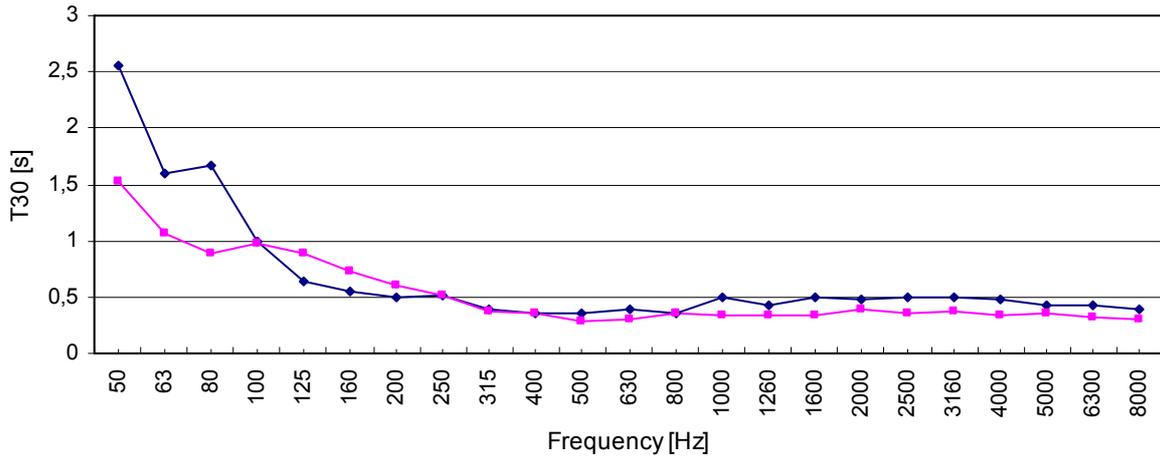


Figure 1: Reverberation time of room 1 (—●—) and room 2 (—■—) measured at the listening position.

To investigate these low-frequency problems, swept-sine measurements were made in both rooms, using one of the monitoring loudspeakers in its usual position and a microphone at the listening/mixing position. Other measurements using different loudspeaker/microphone positions were also realized, to study the spatial variation of the acoustical response and room modes excitation. Figure 2 and 3 represent the acoustical response of room 1 and 2, at the listening position, to a frequency sweep between 50 Hz and 400 Hz. Room 1 shows wide mode spacing mainly below 100 Hz which results from different modes occurring at the same or very close frequencies. A simple theoretical analysis, for an empty rectangular room with rigid walls and similar dimensions, shows modes (2,0,0) and (1,1,0) occurring at 53 Hz, modes (2,0,1) and (1,1,1) at 63 Hz, modes (2,1,1) and (3,0,0) at 79 Hz and modes (3,1,0) and (0,2,0) at 91 Hz. Room 2 has a more regular modal distribution, which may account for the results shown in Figure 1. These two examples are paradigmatic of two possible different approaches that can be used for the design of bass-control devices: either selecting damped resonators tuned to the problematic modal frequencies; or tuning them to different frequencies evenly distributed over a given frequency range.

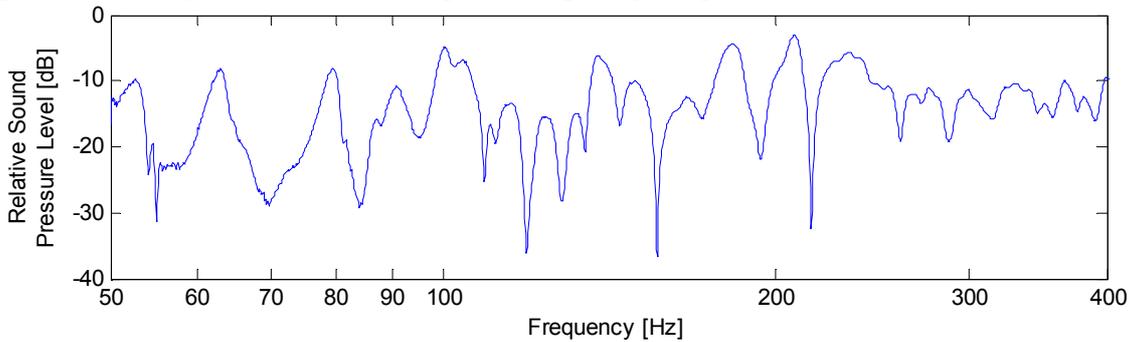


Figure 2: Acoustical response of room 1 at the listening position.

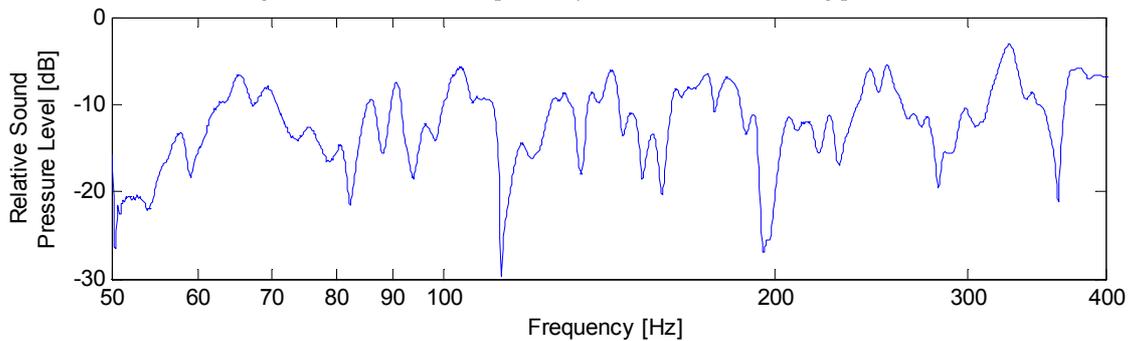


Figure 3: Acoustical response of room 2 at the listening position.

## ACOUSTICAL MODELLING OF THE RESONATORS

The sound propagation model used in this paper is based on the mono-dimensional wave equation, for tubes of variable cross-section  $S(x)$  along their axis. The numerical computation of these continuous systems can be obtained by discretization of the geometry in  $N$  finite conical elements of shape  $S_e(x)$  characterized by a transverse section  $S_1$  at the start of the element and  $S_2$  for the section in the other extremity. For each conical finite element the sound propagation can be described by the Webster equation:

$$\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S_e(x)} \frac{\partial}{\partial x} \left( S_e(x) \frac{\partial p}{\partial x} \right) = 0 \quad (1)$$

The change of pressure inside the element can be described as a linear polynomial of the first order  $p(x,t) = a_0 + a_1 x$ , where the coordinate  $x$  is understood as local (respectively  $x=0$  and  $x=L_e$  in the two nodes of each element). We can derive an approximate solution for  $p(x,t)$ , which satisfies equation (1) in terms of a residual term  $R(x,t)$  to be minimized. Using the Galerkin method we obtain:

$$\int_0^{L_e} R(x,t) N_n(x) dx = 0 \Rightarrow \int_0^{L_e} \{N(x)\} \left[ \frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S_e(x)} \frac{\partial}{\partial x} \left( S_e(x) \frac{\partial p}{\partial x} \right) \right] dx = 0 \quad (2)$$

where  $N_n(x)$  is the weighting function of the spatial approximation and  $\{N(x)\}$  is the corresponding weighting vector derived from the polynomial coefficients. After the necessary integrations, we obtain:

$$[M_e] \{\ddot{P}(t)\} + [K_e] \{P(t)\} = 0 \quad (3)$$

where  $\{P(t)\}$  is the vector describing the pressure at each node of the element. The elementary matrices of mass and rigidity are obtained as

$$[M_e] = \frac{\rho S_1 L_e}{12} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} + \frac{\rho S_2 L_e}{12} \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}; \quad [K_e] = \frac{\rho c^2 (S_1 + S_2)}{2L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (4,5)$$

For the global system, these elementary matrices are assembled as usual. The procedure described in this section allows the computation of the modal frequencies which will be used in the optimization iterations to find the desired shape of the resonator. Although the use of one-dimensional modelling of sound propagation in the resonator is debatable, particularly when cross section dimensions are larger than the wavelengths in question, this approach serves the purpose of this preliminary study. However, three dimensional calculations have already been implemented in this procedure for a different application [7].

## OPTIMIZATION PROCEDURES

Many parameters are involved in a geometry optimization problem, with two unwanted consequences: Firstly, the optimization becomes computationally intensive, and this is further true as the number of parameters to optimize  $P_p$  ( $p = 1, 2, \dots$ ) increases. Secondly, the error hyper-surface  $\mathcal{E}(P_p)$  where the global minimum is searched will display in general many local minima. In [10] we avoided converging to sub-optimal local minima by using a robust (but greedy) global optimization technique namely simulated annealing [11]. In order to improve the computational efficiency, the global optimization algorithm was coupled with a deterministic local optimization technique [11], to accelerate the final stage of the convergence procedure. Very encouraging results have been obtained, demonstrating the feasibility and robustness of this approach, as well as the potential to address some aspects of musical instrument design. However, a negative side effect was the need for significant

computation times, which seem ill-suited to the optimization of large-scale systems such as, for instance, carillon bells. More recently, we alleviated this problem by significantly reducing the dimension of the search space where optimization is performed [6]. This can be achieved in several ways, by describing the geometrical profiles of the vibrating components in terms of a limited number of parameters. Here, we chose to develop  $S(x)$  in terms of a set of orthogonal characteristic functions  $\Psi_s(x)$ , such as Tchebyshev polynomials or trigonometric functions, optimizing their amplitude coefficients. For complex systems, described by finite-element meshes with hundreds or thousands of elements, this approach reduces the size of the optimization problem by several orders of magnitude. Then, we have found that, most often, acceptable solutions can be obtained using efficient local optimization algorithms, leading to a further reduction in computation times. The examples presented in this paper have been obtained using such approach, as described in [6].

In an optimization problem the objective is generally to find the values of a set of variables describing a system that maximizes or minimizes a chosen error function, usually satisfying a set of imposed restrictions. In the present case, we wish to find the optimal shape of the resonator, described by its variable cross section  $S(x)$  and length  $L$  which minimizes the deviations from the computed modal frequencies  $\omega_m[S_e(x), L]$  and the reference target set  $\omega_m^{ref}$ . This error function will be formulated as:

$$\mathcal{E}[S_e(x), L] = \sum_{m=1}^M W_m \left\{ 1 - \frac{\omega_m[S_e(x), L]}{\omega_m^{ref}} \right\}^2 \quad (6)$$

where  $W_m$  are weighting factors for the relative modal errors and  $M$  is the number of modes to optimize.

## OPTIMIZATION RESULTS

In this section we present some examples of resonators of circular section, optimized using the technique of geometric description in terms of characteristic functions coupled with a deterministic optimization algorithm with constraints. The optimizations were carried for two sets of modal frequencies. The first set consisted of 5 frequencies corresponding to the first 5 acoustic modes of Room 1 appearing in Figure 2 (between 50 Hz and 100 Hz). The second set consisted of 10 frequencies distributed logarithmically over the entire frequency range analysed (50 Hz to 400 Hz). The frequencies chosen are described in Table 1.

Table 1: Sets of target modal frequencies.

	Mode	1	2	3	4	5	-	-	-	-	-
Set 1	$f_m$ [Hz]	53.00	63.07	78.97	91.16	100.17	-	-	-	-	-
	$f_m/f_1$	1.00	1.19	1.49	1.72	1.89	-	-	-	-	-
	Mode	1	2	3	4	5	6	7	8	9	10
Set 2	$f_m$ [Hz]	50.00	63.00	79.37	100.00	125.99	158.74	200.00	251.98	317.48	400.00
	$f_m/f_1$	1.00	1.26	1.59	2.00	2.52	3.17	4.00	5.04	6.35	8.00

Figure 4 shows the results of the optimization procedure for Set 1 of modal frequencies using either Cosine functions (a) or Tchebyshev polynomials (b). Although the target modal frequencies are the same, optimization is achieved with somewhat different open-open resonator shapes. From the various geometrical constraints used in these calculations, a maximum and minimum diameter  $D_{max}=50$  cm and  $D_{min}=10$  cm, as well as a maximum resonator length  $L_{max}$  (varying from 1 m to 3.5 m) were used.

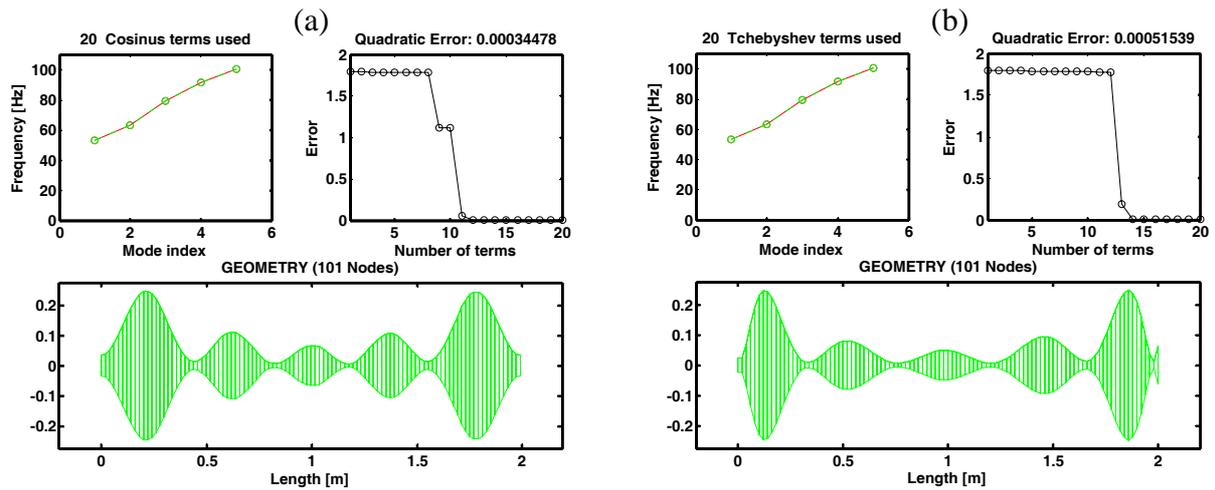


Figure 4: Resonators optimized to Set 1 using (a) Cosine functions or (b) Tchebyshev polynomials.

Although the two resonator shapes are similar, the corresponding acoustic modes can take slightly different forms, as seen in Figures 5 and 6.

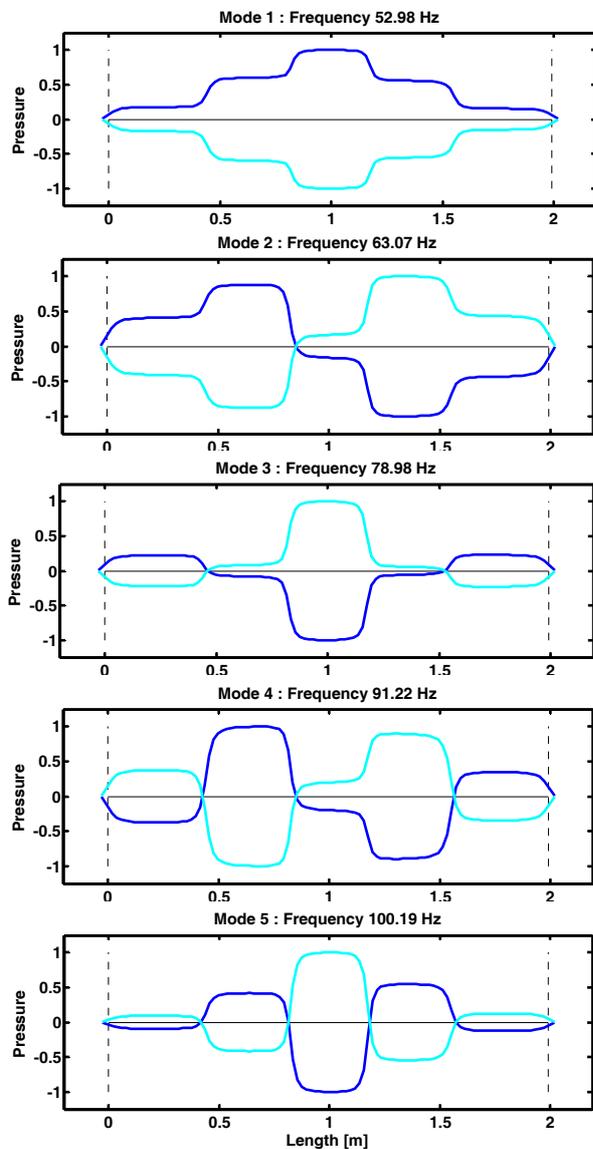


Figure 5: First 5 acoustic mode shapes of resonator (a) in Figure 4.

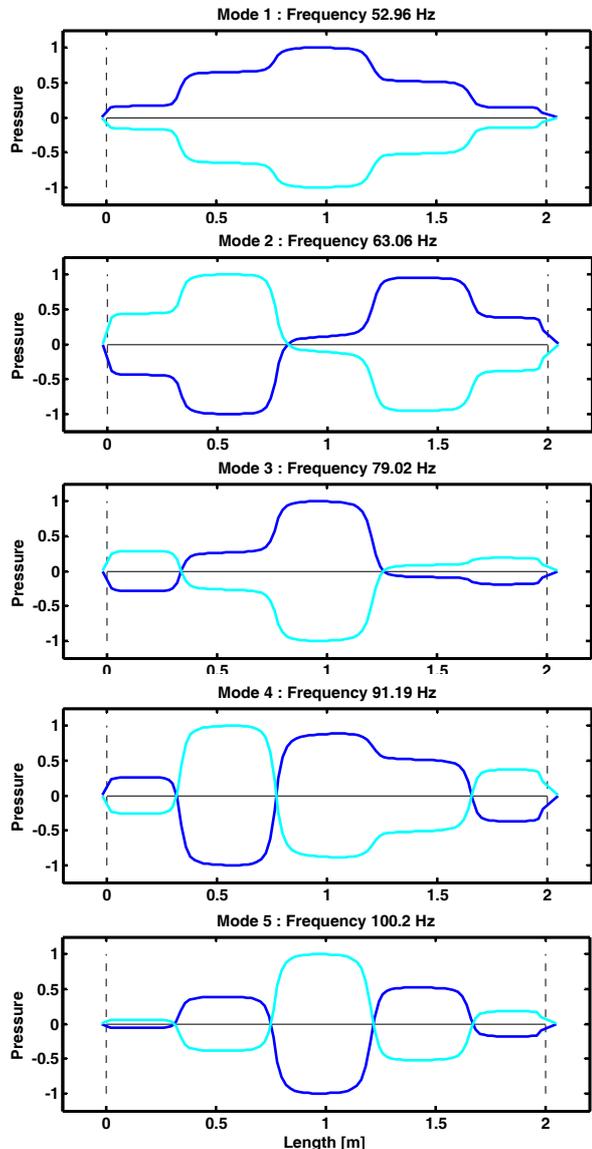


Figure 6: First 5 acoustic mode shapes of resonator (b) in Figure 4.

Figure 7 shows the convergence of the optimization procedure for Set 1 of modal frequencies as the number of characteristic functions (cosines) for the shape description is increased (in odd number of terms). In this example, convergence is obtained after 11 shape functions are used.

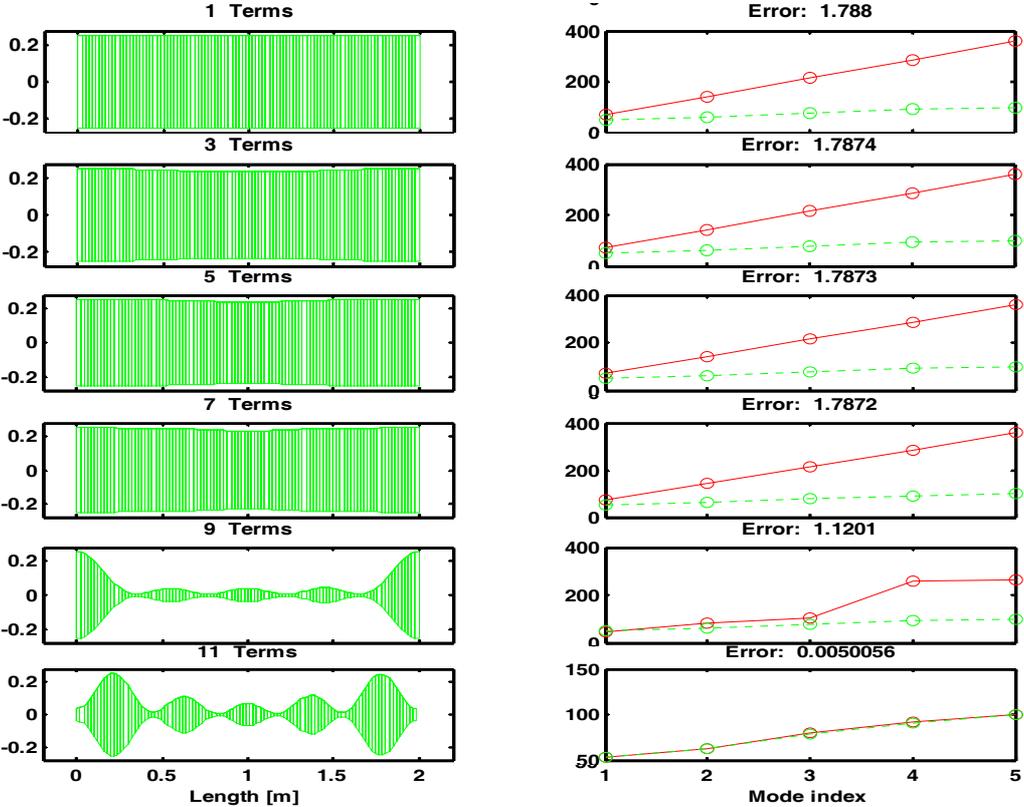


Figure 7: Optimized resonator for the frequency Set 1 (1:1.19:1.49:1.72:1.49). Convergence of the optimization process with the increase of the number of characteristic functions used.

From Figure 4 and 7 one may notice that, quite often, convergence of the results is not gradual but increases by “steps”, as the number of characteristic functions is increased. Figure 8 shows the results of the optimization procedure for Set 2 of modal frequencies using either Cosine functions (a) or Tchebyshev polynomials (b).

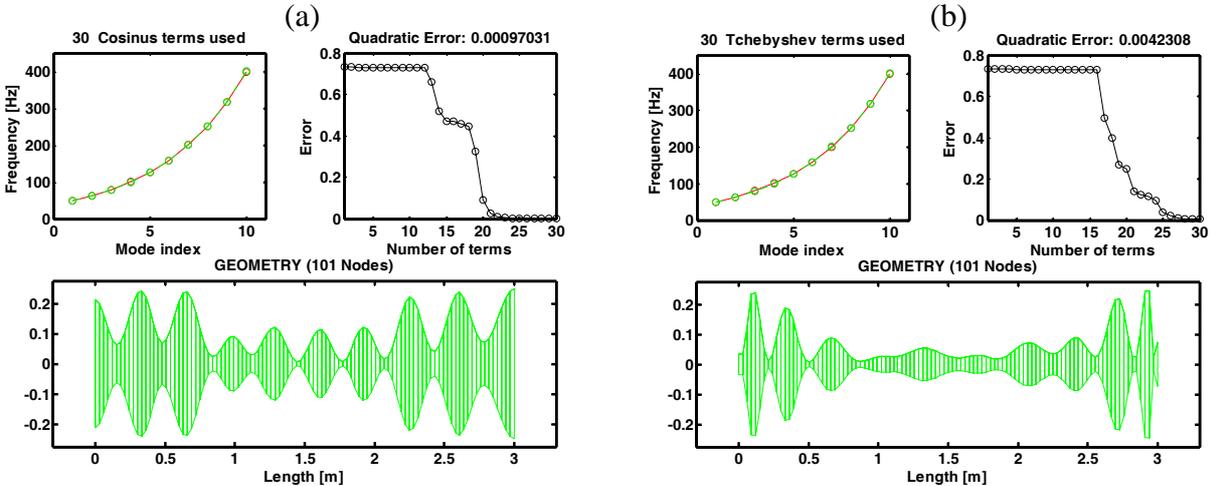


Figure 8: Resonators optimized to Set 2 using (a) Cosine functions or (b) Tchebyshev polynomials.

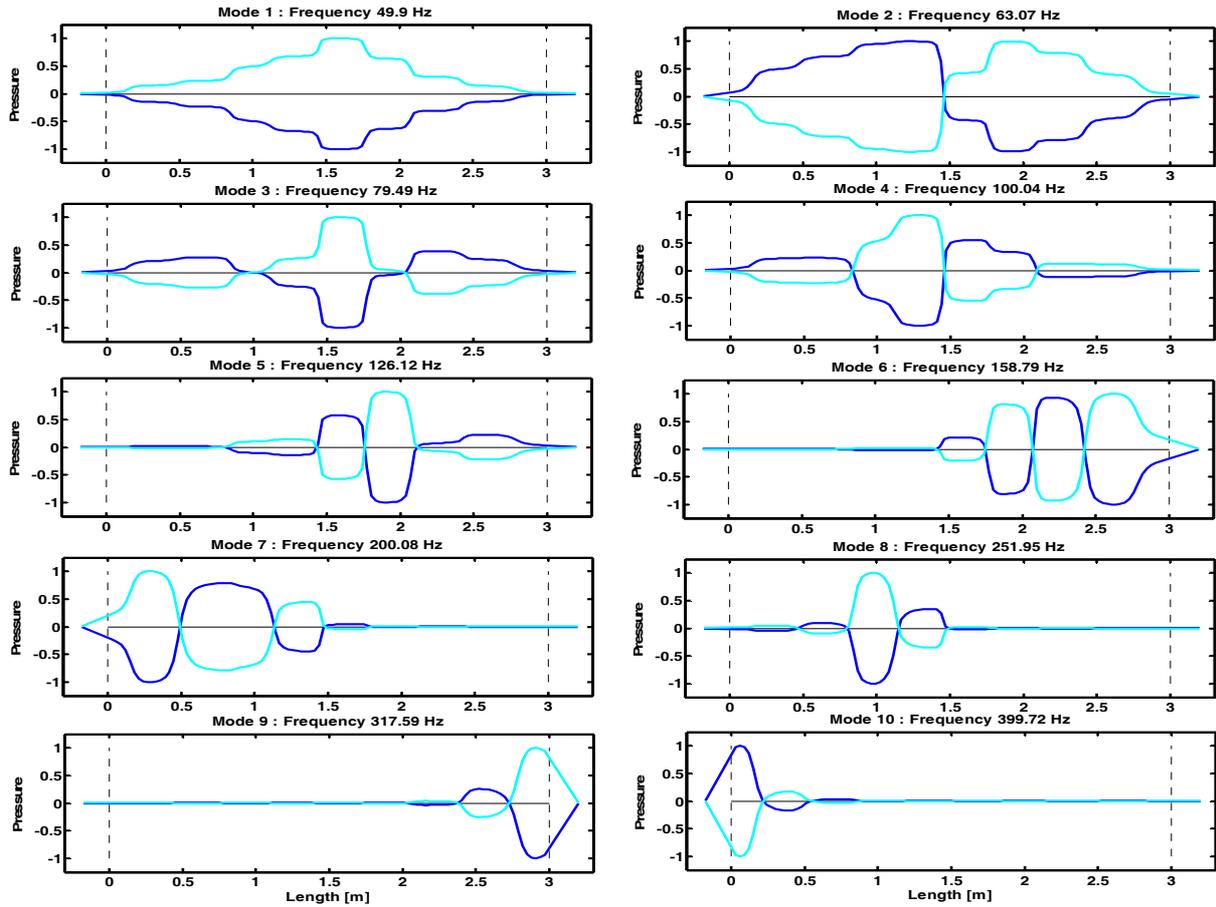


Figure 9: First 10 acoustic mode shapes of resonator (a) in Figure 8.

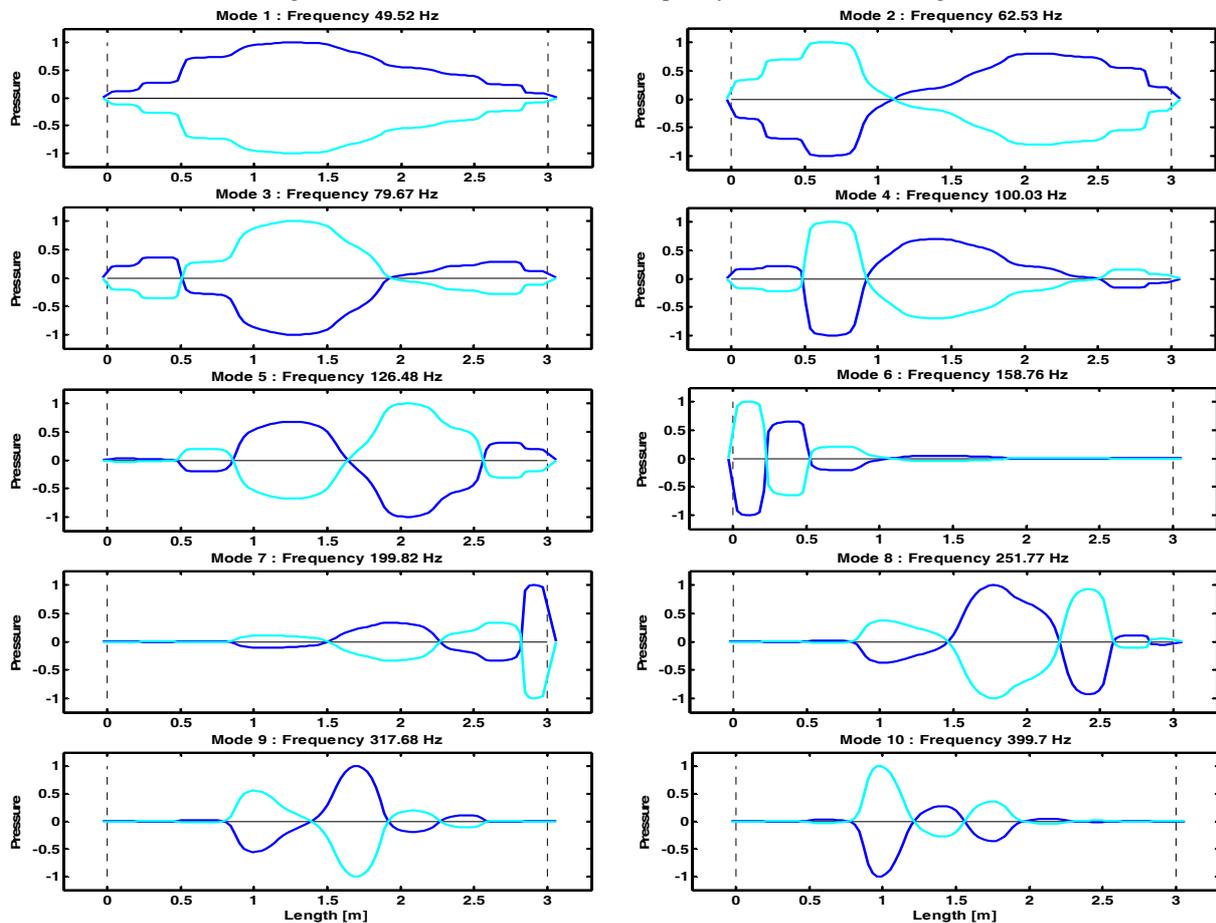


Figure 10: First 10 acoustic mode shapes of resonator (b) in Figure 8.

Although the maximum length might seem very high, it is within the adequate dimensions for a regular control room, depending on the position chosen to install the resonator. Understandably, the demanding target modal frequencies chosen for Set 1 and particularly Set 2, required the use of the full dimensions allowed. However, while for Set 1 a length of 2 m was enough to obtain a negligible error, it took a length of 3 m for a similar satisfactory result for Set 2. The modal errors obtained are presented in Table 2. In all the computations performed for the examples presented here and for other exploratory calculations performed, the optimization made use of the whole resonator length, and both maximum and minimum diameter values. The number of characteristic functions needed for the optimization process is also proportional to the difficulty of the problem, i.e., if the goal proposed comprises a great number of modal frequencies such as in Set 2, the number of shape functions used to obtain a negligible error is also higher. For example, the result of Figure 4(a) was obtained after using only 11 Cosine functions, while for Figure 8(a) it took 21 Cosine functions to reach a similar error. Notice that, for higher frequency modes, the acoustic activity tends to become localized, with each subsystem behaving more independently (see Figures 9 and 10). Also notice that for the optimized resonators of Set 2 identical frequencies are related to quite different modeshapes. It is well known that finding a shape displaying a given set of eigenvalues is a problem which in general presents multiple solutions.

As can also be deduced from inspection of the previous figures, the optimization procedure results in resonator shapes that comprise large volumes connected by short and thin tubes (necks), resembling Helmholtz resonators coupled in series. Interestingly, the number of volumes equals the number of target modal frequencies. However, each mode of the resonator is not particular to one of the volumes and necks as occurs in Helmholtz resonance. On the contrary, each mode shape involves pressure fluctuations over more than one volume and usually extends over the entire resonator. This fact shows that the attempt of designing coupled Helmholtz resonators, in order to achieve broader frequency absorption, based solely on the individual resonances of each component is likely to fail. Although in the simpler case of a double resonator (i.e. two modal frequencies) the use of these devices has been reported as used in the construction of the BBC studios [8]. More recently, these double resonators and their coupling to an enclosure have been thoroughly studied by Doria [9].

All the cases presented so far comprise resonators with both extremities opened. For closed-open resonators, it is more difficult to achieve the right shape for the target frequencies within acceptable geometrical limits and negligible global errors. Figure 11 shows two examples of a closed-open resonator optimized for Set 1 and Set 2 of target modal frequencies, but with less-than-satisfactory errors between the calculated modal frequencies and the target values.

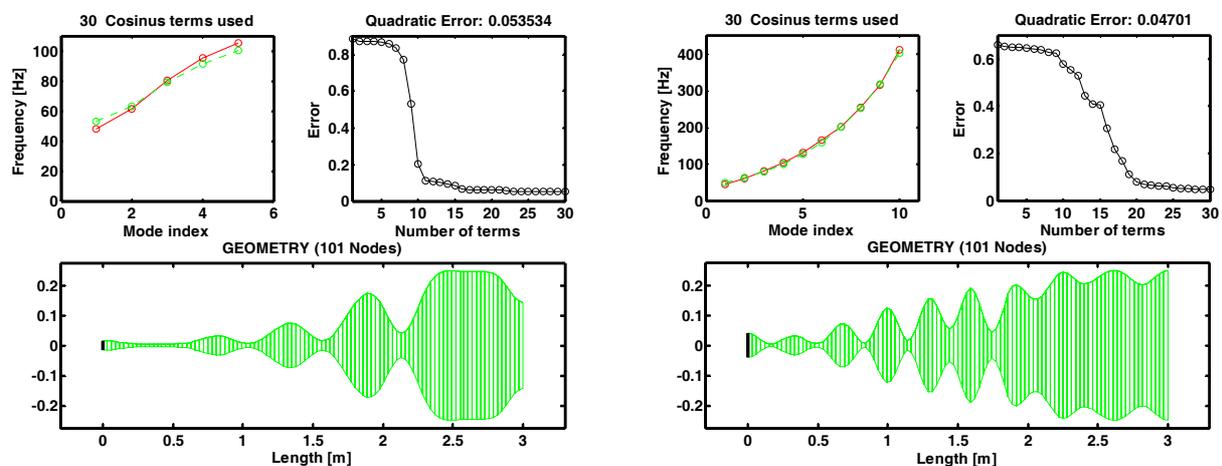


Figure 11: Resonators optimized to (a) Set 1 and (b) Set 2 using Cosine functions.

Table 2: Target, calculated modal frequencies and modal errors for the resonators in Figures 4 and 8.

	Mode	1	2	3	4	5	-	-	-	-	-
Figure 4 (a)	$f_m^c$ [Hz]	52.98	63.07	78.98	91.22	100.19	-	-	-	-	-
	$\varepsilon$ [%]	-0.03	0.00	0.02	0.07	0.02	-	-	-	-	-
	Mode	1	2	3	4	5	-	-	-	-	-
Figure 4 (b)	$f_m^c$ [Hz]	52.96	63.06	79.02	91.19	100.2	-	-	-	-	-
	$\varepsilon$ [%]	-0.08	-0.01	0.07	0.03	0.03	-	-	-	-	-
	Mode	1	2	3	4	5	6	7	8	9	10
Figure 8 (a)	$f_m^c$ [Hz]	49.9	63.07	79.49	100.04	126.12	158.79	200.08	251.95	317.59	399.72
	$\varepsilon$ [%]	-0.19	0.11	0.15	0.04	0.1	0.03	0.04	-0.01	0.03	-0.07
	Mode	1	2	3	4	5	6	7	8	9	10
Figure 8 (b)	$f_m^c$ [Hz]	49.52	62.53	79.67	100.03	126.48	158.76	199.82	251.77	317.68	399.7
	$\varepsilon$ [%]	-0.96	-0.75	0.37	0.03	0.39	0.01	-0.09	-0.09	0.06	-0.07

## CONCLUSIONS

In this paper we presented an effective technique for the shape optimization of resonators in order to obtain a target set of modal frequencies characteristic of resonances occurring in regular control rooms. A computational strategy based on a mono-dimensional wave propagation model coupled with a classical gradient-based optimization approach proved very effective. In particular, smooth shapes and very fast optimizations were achieved by using various sets of orthogonal functions for describing the geometry.

Two different approaches have been suggested to tackle with the problem of undesirable low-frequency resonances: (1) exact resonator mode-matching and (2) evenly spaced resonator modes. Optimized designs have been produced for two different control rooms following both strategies. The numerical results are promising and will be followed by experimental work to be reported elsewhere.

## ACKNOWLEDGEMENTS

The authors would like to thank the Audio Services of the Polytechnic Institute of Porto for the access to the control rooms analysed in this study.

## REFERENCES

- [1] H. Helmholtz, *On the Sensation of Tone*, Dover, New York, 1954
- [2] U. Ingard, On the Theory and Design of Acoustic Resonators, *Journal of the Acoustical Society of America*, **25**(6), 1037-1061, 1953
- [3] R. C. Chanaud, Effects of Geometry on the Resonance Frequency of Helmholtz Resonators, *Journal of Sound and Vibration*, **178**(3), 337-348, 1994
- [4] F. J. Fahy and C. Schofield, A Note on the Interaction Between a Helmholtz Resonator and an Acoustic Mode of an Enclosure, *Journal of Sound and Vibration*, **72**(3), 365-378, 1980
- [5] A. Cummings, The Effects of a Resonator Array on the Sound Field in a Cavity, *Journal of Sound and Vibration*, **154**(1), 25-44, 1992
- [6] L. Henrique, J. Antunes, Optimal Design and Physical Modelling of Mallet Percussion Instruments, *Acustica/Acta Acustica*, **89**, 948-963, 2003
- [7] L. Henrique *et al*, Application of optimization techniques for acoustical resonators, Proceedings of the 12<sup>th</sup> International Congress on Sound and Vibration, Lisbon, 2005
- [8] T. Cox and P. D'Antonio, *Acoustic Absorbers and Diffusers – Theory Design and Application*, Spon Press, London, 2004
- [9] A. Doria, Control of Acoustic Vibrations of an Enclosure by means of Multiple Resonators, *Journal of Sound and Vibration*, **181**(4), 673-685, 1995
- [10] L. Henrique, J. Antunes, J. Carvalho, Design of Musical Instruments Using Global Optimization Techniques, Proceedings of the ISMA 2001, Perugia, 2001
- [11] P. Venkataraman, *Applied Optimization with MATLAB® Programming*, Wiley-Interscience, New York, 2002.