Design of duct cross sectional areas in bass-trapping resonators for control rooms

Octávio Inácio,[a] Luís Henrique,[b] and José Antunes[c]

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Small rooms, such as the ones specifically designed for listening to amplified music, like control rooms in recording studios, face the problem of low-frequency over-enhancement by acoustic resonances. Several devices have been developed to tackle this problem, such as Helmoltz resonators. The number of controlled acoustic modes depends on several factors among which are the central frequency chosen, the modal density in that frequency range, and the coupling between the resonator and the room. In this paper we suggest that the efficiency of such resonators may be significantly improved if, instead of using basic Helmholtz or devices with uniform cross-section, more complex shape-optimized resonators are used, in order to cope with a larger number of undesirable acoustic modes. We apply optimization techniques to the uncoupled resonator, developed in our previous work, in order to obtain the optimal shapes for devices that resonate at a design set of acoustic eigenvalues, within imposed physical and/or geometrical constraints. One-dimensional and three-dimensional finite element models were implemented. The one-dimensional model was coupled to optimization techniques in order to achieve the design goal. We illustrate the proposed approach with two examples of resonator shapes and different design sets of absorption frequencies.

Primary subject classification: 34.3; Secondary subject classification: 76.1.2

1 INTRODUCTION

The acoustical design of small rooms for high fidelity sound reproduction requires particular attention to the control low-frequency resonances. The imbalance between over-enhancement of sound at these modal frequencies and the absence of room response at anti-resonances produces a detrimental lack of uniformity of the room acoustic response. This effect is more pronounced for the frequency range where modal density and modal bandwidth (or modal damping) are low. Additionally, the room dimensions may be such that packs of modes occur in certain frequency ranges, not only maximizing the resonance effect but also creating separation between different peaks in the room frequency response.

These and other related problems have been tackled, with more or less efficiency, by the use of Helmholtz resonators, membrane panels or tube-traps, among many others. The uncoupled resonance behaviour of these bass control devices is typically focused on a central frequency of maximum sound absorption which spreads over a determined bandwidth. The number of controlled acoustic modes depends on several factors among which are the central resonance frequency chosen, the modal density in the controlled frequency range, damping, and the ratio of the resonator to room volumes (see Ref. 1 for further discussion). The degree of attenuation of the resonance effect is dependent not only on the number of such devices used, but also on their location in the room, ideally close to pressure antinodes of the controlled mode. Helmholtz resonators have been particularly used in many different applications where an accurate control of a single frequency is desired. These resonators have been thoroughly studied since the 19th century beginning with the work of Helmholtz.[2] More recently, several

a) Musical Acoustics Laboratory, Escola Superior de Música e das Artes do Espectáculo do Instituto Politécnico do Porto, Rua da Alegria, 503, 4000-045 Porto, PORTUGAL; email: OctavioInacio@esmae-ipp.pt

b) Musical Acoustics Laboratory, Escola Superior de Música e das Artes do Espectáculo do Instituto Politécnico do Porto, Rua da Alegria, 503, 4000-045 Porto, PORTUGAL; email: LuisHenrique@esmae-ipp.pt

c) Applied Dynamics Laboratory, Instituto Tecnológico e Nuclear, 2686 Sacavém codex, PORTUGAL; email: jantunes@itn.pt

researchers became interested in the optimization of the design and physical behaviour of such systems,[3] on the effect of basic geometry on changing on the resonant frequency,[4,5] and on the acoustical coupling between the resonator and the room,[1,6] to mention a few.

In this paper we suggest that the efficiency of such resonators may be significantly improved if, instead of using basic Helmholtz resonators or devices with uniform cross-section, more complex shape-optimized resonators be used, in order to cope with a larger number of undesirable acoustic modes. We apply optimization techniques recently developed in our previous work,[7,8] in order to obtain optimal shapes for such devices so that they resonate at a design set of acoustic eigenvalues, within imposed physical and/or geometrical constraints. A simple 1D finite element acoustic model was implemented and coupled with optimization techniques in order to achieve this goal with fast computations. We illustrate the proposed approach with several examples of resonator shapes and different design sets of absorption frequencies. Then we discuss the validity of the simple 1D acoustic model in the context of the present application, by performing more involved 3D finite element model acoustic computations on a few optimized resonators.

For this preliminary analysis we will focus only on the modal behaviour of the resonator isolated from the room. However, the complete analysis of this problem has to consider the frequency shifts and room mode shape distortion arising from the acoustical coupling between the room and the resonator. Additionally, viscous boundary layer absorption effects which account for the damping at the neck of the resonators were not addressed in this model. These aspects will be addressed elsewhere.

2 EXPERIMENTAL ANALYSIS OF TWO CONTROL ROOMS

In order to obtain realistic examples of problematic acoustical resonance effects, two different control rooms were experimentally analysed. These control rooms belong to the College of Music and Performing Arts of the Polytechnic Institute of Porto, and are aimed to support the work of students of the Production and Music Technologies Degree, as well as the development of professional work by the Institute Audio Services. Both rooms have received acoustical treatment for the medium and high frequency range but have considerable problems in the reverberation time below 200 Hz. Figure 1 presents the results of reverberation time measurements carried out in both rooms using the monitor loudspeakers located on the mixing table and a microphone at the listener/mixing position.

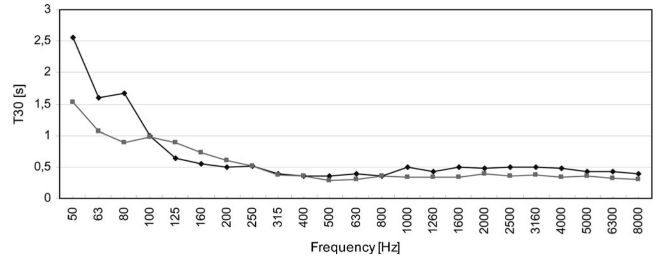

Fig. 1—Reverberation time of room 1 (—) and room 2 (—) measured at the listening position.

To investigate these low-frequency problems, swept-sine measurements were made in both rooms, using one of the monitoring loudspeakers in its usual position and a microphone at the listening/mixing position. Other measurements using different loudspeaker/microphone positions were also realized, to study the spatial variation of the acoustical response and room modes excitation. Figs. 2 and 3 represent the acoustical response of room 1 (6.47 m33.75 m34.65 m) and room 2 (7.5 m33 m34.56 m), at the listening position, to a frequency sweep between 50 Hz and 400 Hz. Room 1 shows wide resonance spacing, mainly below 100 Hz, with several mode packets which results from different modes occurring in that frequency range. Indeed, a simple theoretical analysis, for an empty rectangular room with rigid walls and similar dimensions, shows modes (2,0,0) and (1,1,0) occurring at approximately 53 Hz, modes (2,0,1) and (1,1,1) at 64 Hz, modes (2,1,1) and (3,0,0) at 79 Hz and modes (3,1,0) and (0,2,0) at 91 Hz. Room 2 has a more regular modal distribution, which may account for the lower reverberation times shown in Fig. 1 for room 2. These

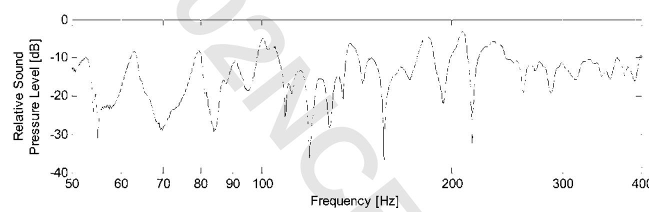

Fig. 2—Acoustical response of room 1 at the listening position.

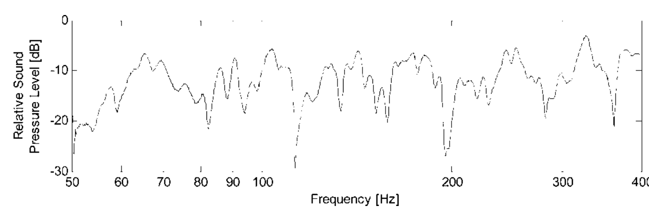

Fig. 3—Acoustical response of room 2 at the listening position.

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two examples are paradigmatic of two possible different approaches that can be used for the design of bass-control devices: either selecting damped resonators tuned to the problematic modal frequencies; or tuning them to different frequencies evenly distributed over a given frequency range.

3 ACOUSTICAL MODELLING OF THE RESONATORS

In order to allow for very fast computations, the sound propagation model used for the optimization procedure in this paper is based on the one-dimensional wave equation approximation, for tubes of variable cross-section $S(x)$ along their axis. The numerical computation of these continuous systems can be obtained by discretization of the geometry in N finite conical elements of section $S_e(x)$ characterized by a transverse section S_1 at the start of the element and S_2 for the section in the other extremity. For each conical finite element the sound propagation can be described by the Webster equation:

$$\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S_e(x)} \frac{\partial}{\partial x}\left(S_e(x) \frac{\partial p}{\partial x} \right) = 0 \tag{1}$$

The change of pressure inside the element can be described as a linear first order polynomial $p(x,t)=a_0 + a_1 x$, where the coordinate x is understood as local (respectively $x=0$ and $x=L_e$ at the two nodes of each element). We can derive an approximate solution for $p(x,t)$, which satisfies Eqn. (1) in terms of a residual term $R(x,t)$ to be minimized, using the Galerkin method:

$$\begin{aligned} &\int_0^L R(x,t) N_n(x) dx \\ &= 0 \Rightarrow \int_0^{L_e} \{N(x)\} \left[\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S_e(x)} \frac{\partial}{\partial x}\left(S_e(x) \frac{\partial p}{\partial x} \right) \right] dx \\ &= 0 \end{aligned} \tag{2}$$

where $N_n(x)$ is the weighting function of the spatial approximation and $\{N(x)\}$ is the corresponding weighting vector derived from the polynomial coefficients. After the necessary integrations, we obtain:

$$[M_e]\{\ddot{P}(t)\} + [K_e]\{P(t)\} = 0 \tag{3}$$

where $\{P(t)\}$ is the vector describing the pressure at each node of the element. The elementary matrices of $[M_e]$ for mass and $[K_e]$ for rigidity are obtained as:

$$[M_e] = \frac{\rho S_1 L_e}{12} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} + \frac{\rho S_2 L_e}{12} \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}; \tag{4}$$

$$[K_e] = \frac{\rho c^2 (S_1 + S_2)}{2 L_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{5}$$

For the global system, these elementary matrices are assembled as usual. The procedure described in this section allows the computation of the system modal frequencies, which will be used in the optimization iterations to find the desired shape of the resonator.

The use of 1D modelling of sound propagation for resonator design may be justified or debatable, depending on the relative magnitude of the gradient component along the radial direction of the real pressure field $\partial p(r,x,t)/\partial r$. For frequencies sufficiently high, such is the case when wavelengths become of the order of magnitude or smaller than the resonator diameter. At lower frequencies, it is well known that the Webster equation–and hence 1D finite element modelling–can be safely adopted, provided that the cross-section $S(x)$ changes smoothly along the resonator axis. However, simple 1D modelling may be ill suited, if the axial change of the cross-section $\partial S(x)/\partial x$ is not smooth. This issue will be further expanded on later in the paper.

As can be seen, no damping term is included in the previous formulation. Although the damping term is very important when considering the absorption efficiency of these devices, its effect in the calculated resonance frequencies is only marginal, and is therefore neglected for the scope of this work.

4 OPTIMIZATION PROCEDURES

Many parameters are involved in a geometry optimization problem, with two unwanted consequences. Firstly, the optimization becomes computationally intensive, and this is further true as the number of parameters to optimize P_p $(p=1,2,\ldots)$ increases. Secondly, the error hyper-surface $\varepsilon(P_p)$ where the global minimum is searched will display in general many local minima. In Ref. 9 we avoided converging to sub-optimal local minima by using a robust (but greedy) global optimization technique, namely simulated annealing.[10] In order to improve the computational efficiency, the global optimization algorithm was coupled with a deterministic local optimization technique,[10] to accelerate the final stage of the convergence procedure. Very encouraging results have been obtained, demonstrating the feasibility and robustness of this approach, as well as its potential to address some aspects of musical instrument design. However, a negative side effect was the need for significant computation times, which seem ill-suited for the optimization of large-scale systems such as, for instance, carillon bells. More recently, we alleviated this problem by significantly reducing the dimension of the search

Table 1—Sets of target modal frequencies.

Set 1	Mode	1	2	3	4	5	-	-	-	-	-
	f_m [Hz]	53.00	63.07	78.97	91.16	100.17	-	-	-	-	-
	f_m/f_1	1.00	1.19	1.49	1.72	1.89	-	-	-	-	-
Set 2	Mode	1	2	3	4	5	6	7	8	9	10
	f_m [Hz]	50.00	63.00	79.37	100.00	125.99	158.74	200.00	251.98	317.48	400.00
	f_m/f_1	1.00	1.26	1.59	2.00	2.52	3.17	4.00	5.04	6.35	8.00

space where optimization is performed.[7] This can be achieved in several ways, by describing the geometrical profiles of the vibrating components in terms of a limited number of parameters. Here, we chose to develop $S(x)$ in terms of a set of orthogonal characteristic functions $\Psi_s(x)$, such as Tchebyshev polynomials or trigonometric functions, and then optimizing their amplitude coefficients. For complex systems, described by finite-element meshes with hundreds or thousands of elements, this approach reduces the size of the optimization problem by several orders of magnitude. Then, we have found that, most often, acceptable solutions can be obtained using efficient local optimization algorithms, leading to a further reduction in computation times. The examples presented in this paper have been obtained using such approach, as described in Ref. 7.

In an optimization problem the objective is generally to find the values of a set of variables describing a system that maximizes or minimizes a chosen error function, usually satisfying a set of imposed restrictions. In the present case, we wish to find the optimal shape of the resonator, described by its variable cross section $S(x)$ and length L which minimizes deviations between the computed modal frequencies $\omega_m[S_e(x),L]$ and the reference target set ω_m^{ref}. This error function will be formulated as:

$$\varepsilon[S_e(x),L] = \sum_{m=1}^{M} W_m \left\{ 1 - \frac{\omega_m[S_e(x),L]}{\omega_m^{ref}} \right\}^2 \qquad (6)$$

where W_m are weighting factors for the relative modal errors and M is the number of modes to optimize.

5 OPTIMIZATION RESULTS

In this section we present some examples of resonators of circular section, optimized using the previously described technique of geometric description in terms of characteristic functions coupled with a deterministic optimization algorithm with constraints. The optimizations were carried for two sets of modal frequencies. The first set consisted of 5 frequencies corresponding to the first 5 acoustic modes of Room 1 appearing in Fig. 2 (between 50 Hz and 100 Hz). The second set consisted of 10 frequencies distributed logarithmically over the entire frequency range analysed (50 Hz to 400 Hz). The frequencies chosen are described in Table 1.

Figure 4 shows the results of the optimization procedure for Set 1 of modal frequencies using either Cosine functions (a) or Tchebyshev polynomials (b). Although

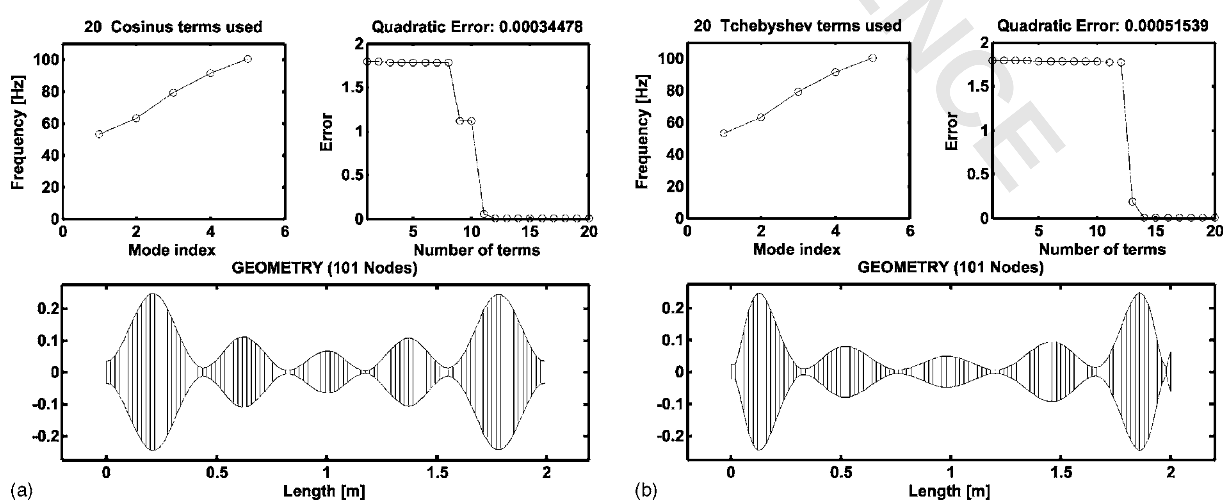

Fig. 4—Resonators optimized to Set 1 using (a) Cosine functions or (b) Tchebyshev polynomials.

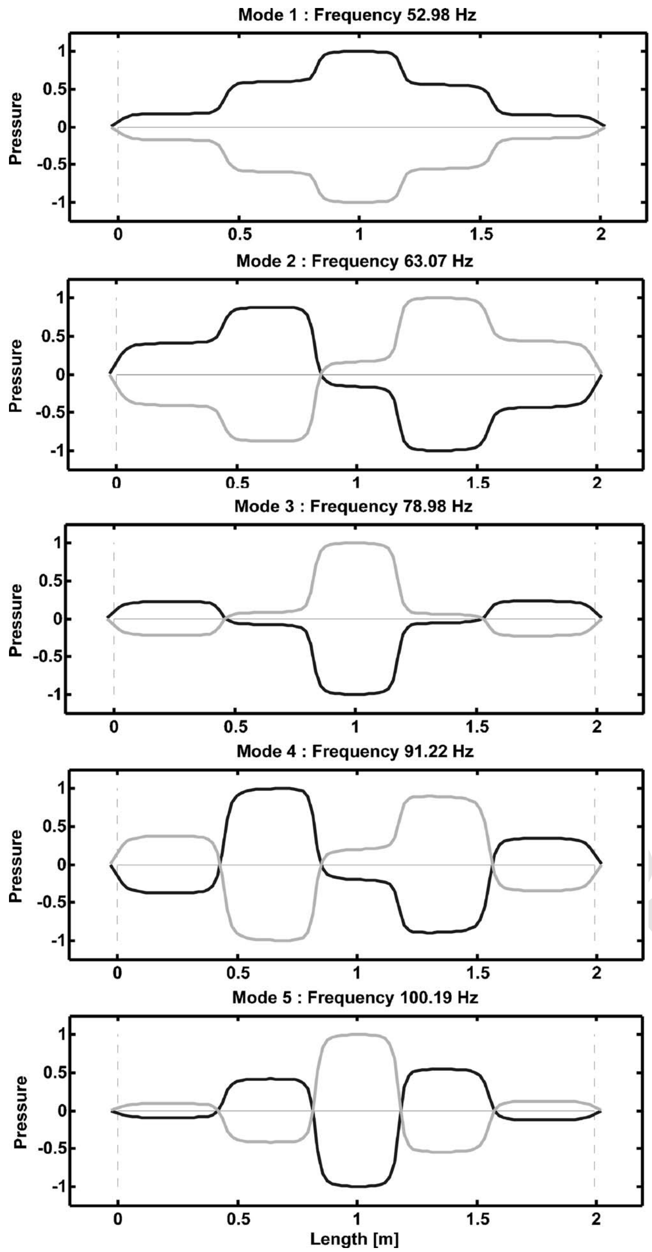

Fig. 5—First 5 pressure modes shapes of resonator (a) in Figure 4.

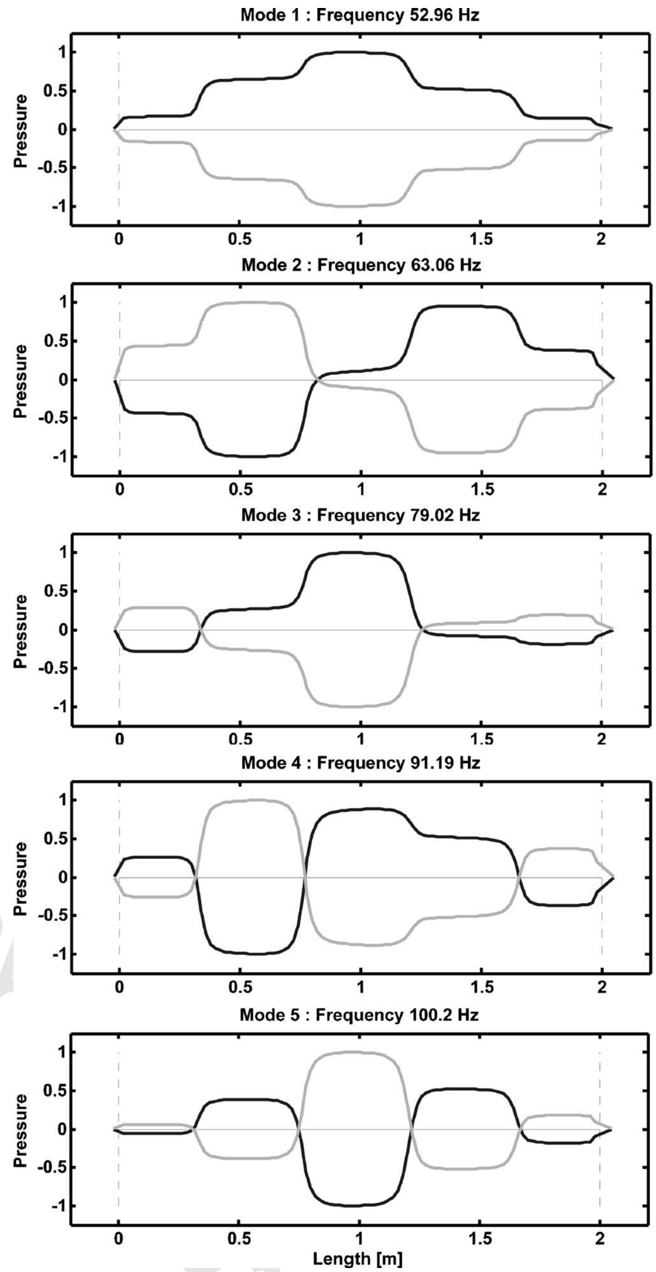

Fig. 6—First 5 pressure mode shapes of resonator (b) in Figure 4.

the target modal frequencies are the same, optimization is achieved with somewhat different open-open resonator shapes. From the various geometrical constraints used in these calculations, a maximum and minimum diameter $D_{max}=50$ cm and $D_{min}=10$ cm, as well as a maximum resonator length L_{max} (varying from 1 m to 3.5 m) were imposed.

Although the two resonator shapes are similar, the corresponding acoustic mode shapes can take slightly different forms, as seen in Figs. 5 and 6.

Figure 7 shows the convergence of the optimization procedure for Set 1 of modal frequencies as the number of characteristic functions (cosines) for the shape description is increased (in odd number of terms). For each iteration, the left-hand graph represents the shape of the resonator, while the right-hand graph displays the target modal frequencies (light dotted line) and the current modal frequencies (black full line). In this example, convergence is obtained after 11 characteristic functions are used.

From Figs. 4 and 7 one may notice that, quite often, convergence of the results is not gradual but increases by "steps", as the number of characteristic functions is increased. Fig. 8 shows the results of the optimization procedure for Set 2 of modal frequencies using either Cosine functions (a) or Tchebyshev polynomials (b).

Although the maximum length might seem high, it is

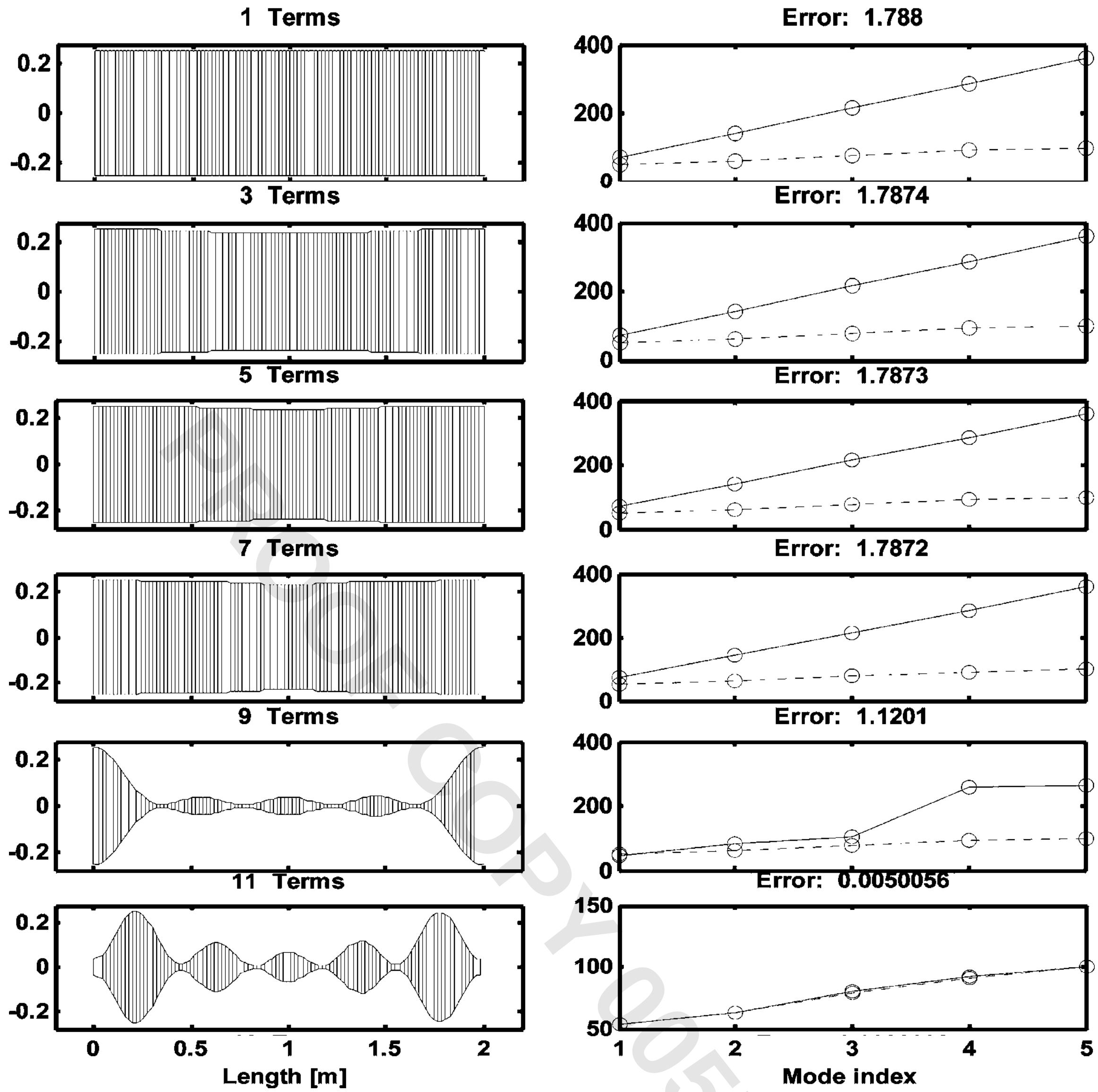

Fig. 7—Optimized resonator for the frequency Set 1 (1:1.19:1.49:1.72:1.49). Convergence of the optimization process with the increase of the number of characteristic functions used.

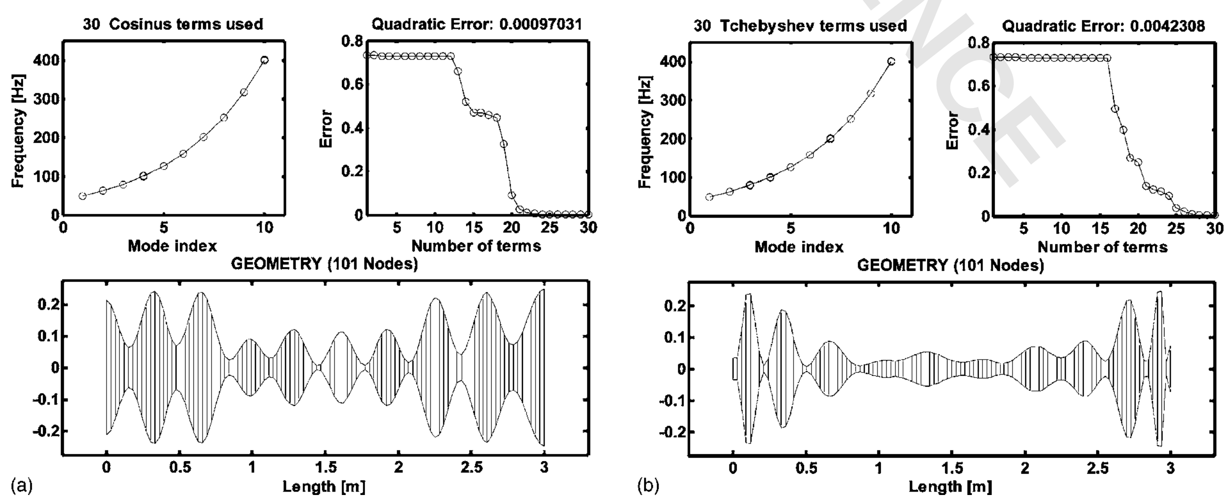

Fig. 8—Resonators optimized to Set 2 using (a) Cosine functions or (b) Tchebyshev polynomials.

Table 2—Target, calculated modal frequencies and modal errors for the resonators in Figures 4 and 8.

Figure 4(a)	Mode	1	2	3	4	5	-	-	-	-	-
	f_m^c [Hz]	52.98	63.07	78.98	91.22	100.19	-	-	-	-	-
	ε [%]	−0.03	0.00	0.02	0.07	0.02	-	-	-	-	-
Figure 4(b)	Mode	1	2	3	4	5	-	-	-	-	-
	f_m^c [Hz]	52.96	63.06	79.02	91.19	100.2	-	-	-	-	-
	ε [%]	−0.08	−0.01	0.07	0.03	0.03	-	-	-	-	-
Figure 8(a)	Mode	1	2	3	4	5	6	7	8	9	10
	f_m^c [Hz]	49.9	63.07	79.49	100.04	126.12	158.79	200.08	251.95	317.59	399.72
	ε [%]	−0.19	0.11	0.15	0.04	0.1	0.03	0.04	−0.01	0.03	−0.07
Figure 8(b)	Mode	1	2	3	4	5	6	7	8	9	10
	f_m^c [Hz]	49.52	62.53	79.67	100.03	126.48	158.76	199.82	251.77	317.68	399.7
	ε [%]	−0.96	−0.75	0.37	0.03	0.39	0.01	−0.09	−0.09	0.06	−0.07

within the adequate dimensions for a regular control room, depending on the position chosen to install the resonator. Understandably, the target modal frequencies chosen for Set 1 and particularly Set 2, required the use of the full dimensions allowed. However, while for Set 1 a length of 2 m was enough to obtain a negligible error, it took a length of 3 m for a similar satisfactory result for Set 2. The modal errors obtained are presented in Table 2. In all the computations performed for the examples presented here and for other exploratory calculations performed, the optimization made use of the whole resonator length, and both maximum and minimum diameter values. The number of characteristic functions needed for the optimization process is also proportional to the difficulty of the problem, i.e., if the goal comprises a great number of modal frequencies such as in Set 2, the number of characteristic functions used to obtain a negligible error is also higher. For example, the result of Fig. 4(a) was obtained after using only 11 Cosine functions, while for Fig. 8(a) it took 21 Cosine functions to reach a similar error. Notice that, for higher frequency modes, the acoustic activity tends to become localized, with each subsystem behaving more independently (see Figs. 9 and 10). Also notice that for the optimized resonators of Set 2 identical frequencies are related to quite different modeshapes. It is well known that finding the system shape leading to a given set of eigenvalues is a problem which in general presents multiple solutions.

As can also be deducted from inspection of the previous figures, the optimization procedure results in resonator shapes that comprise large volumes connected by short and thin tubes (necks), resembling Helmholtz resonators coupled in series. Interestingly, the number of volumes equals the number of target modal frequencies. However, each mode of the resonator is not coupled to just one of the volumes and necks as occurs in Helmholtz resonance. On the contrary, each mode shape involves pressure fluctuations over more than one volume and usually extends over the entire resonator. This fact shows that the attempt to design coupled Helmholtz resonators, in order to achieve broader frequency absorption, based solely on the individual resonances of each component is likely to fail, although in the simpler case of a double resonator (i.e. two modal frequencies) the use of these devices has been reported as used in the construction of the BBC studio.[11] More recently, these double resonators and their coupling to an enclosure have been thoroughly studied by Doria.[12]

All the cases presented so far comprise resonators with both extremities opened. For closed-open resonators and the target-set modal frequencies of the two examples in this paper, we found it is more difficult to achieve the right shape for the target frequencies within acceptable geometrical limits and negligible global errors. Fig. 11 shows two examples of a closed-open resonator optimized for Set 1 and Set 2 of target modal frequencies, but with less-than-satisfactory errors between the calculated modal frequencies and the target values.

6 REFINED ACOUSTICAL MODELLING

As discussed earlier, the simple and fast 1D acoustical model should be limited to lower frequency modes and resonator geometries with relatively smooth changes in cross-section. In this section we will briefly

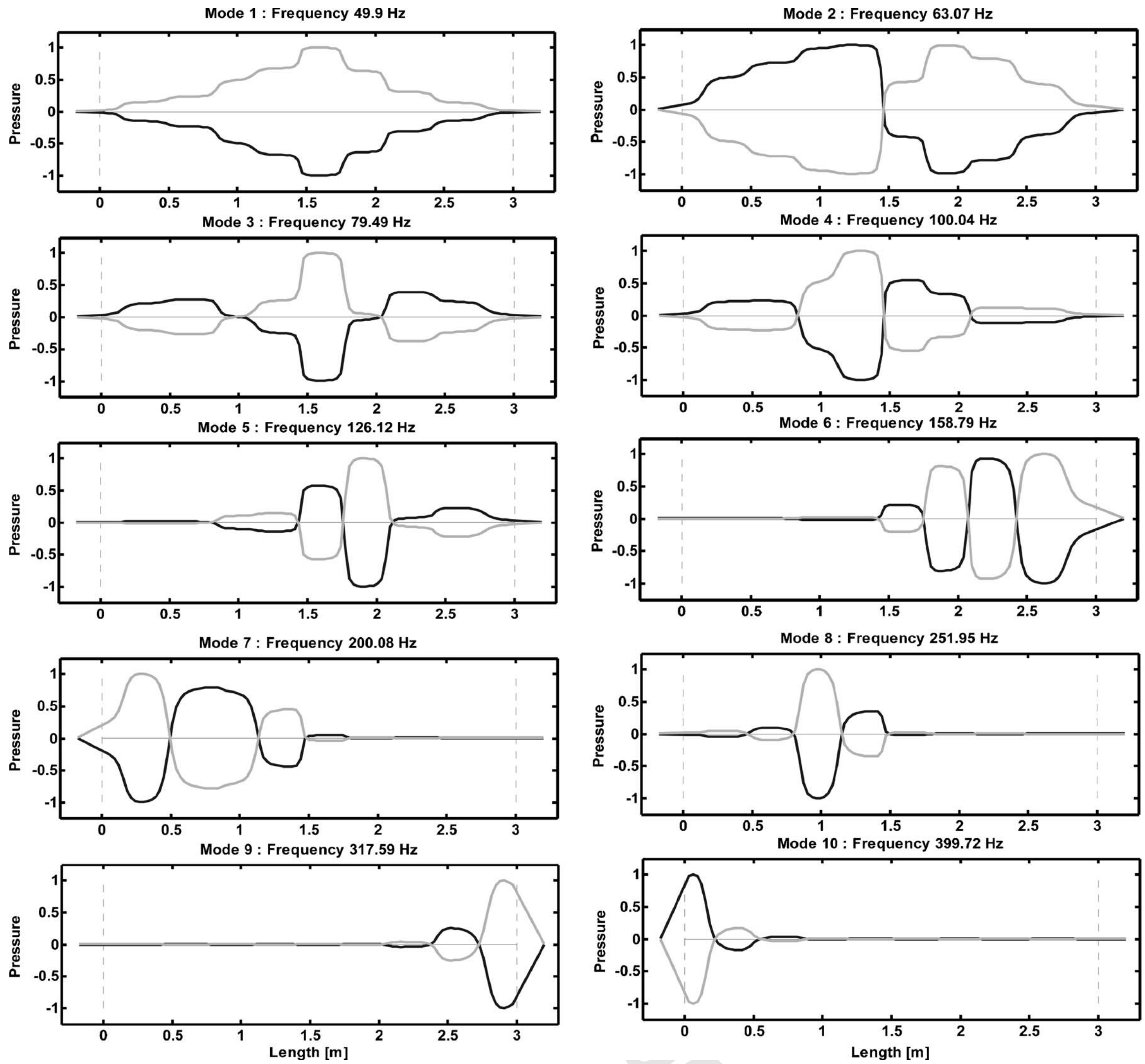

Fig. 9—First 10 pressure mode shapes or resonator (a) in Figure 8.

illustrate this issue, in connection with the present problem, by re-computing the acoustical modes of the optimized resonators shown in Fig. 8, using now a full 3D finite element model for the wave equation:

$$\ddot{p}(r,x,t) - c^2\nabla^2 p(r,x,t) = 0 \qquad (7)$$

Each computed domain was discretized using tetrahedral acoustic elements, applying a boundary condition $\partial p/\partial r|_{\partial R}=0$ at the resonator wall. Two additional external volumes have been included, extending the open extremities of the resonators, which were able to emulate realistically the modal sound field at flanged open extremities, for the first 6 modes computed. At the external boundaries of the additional volumes, the boundary condition $p|_{\partial V}=0$ was used.

As a compact illustration, Fig. 12 displays the computed acoustical mode shapes of the first six modes of the optimized resonator geometry shown in Fig. 8(a). Comparison between these modeshapes and those shown in Fig. 9 reveals a remarkable consistence, indicating that the simple 1D computations lead to a good qualitative agreement. However, quantitative results are not so flattering and it is important to stress that the modal frequencies stemming from the 3D computations were consistently lower than those produced by the simple model, with differences ranging from 5% up to about 20% in the frequency range of interest. It is worth mentioning that, for the somewhat smoother geometries obtained by the authors in the context of a different application,[8] such errors were within 3%. However, for resonators with geometries such as those addressed in this paper, the over-estimation errors in modal frequencies seems excessive for most practical designs, pointing the need for more refined acoustical modelling when dealing with real applications.

7 CONCLUSIONS

In this paper we presented an effective technique for the shape optimization of resonators in order to obtain

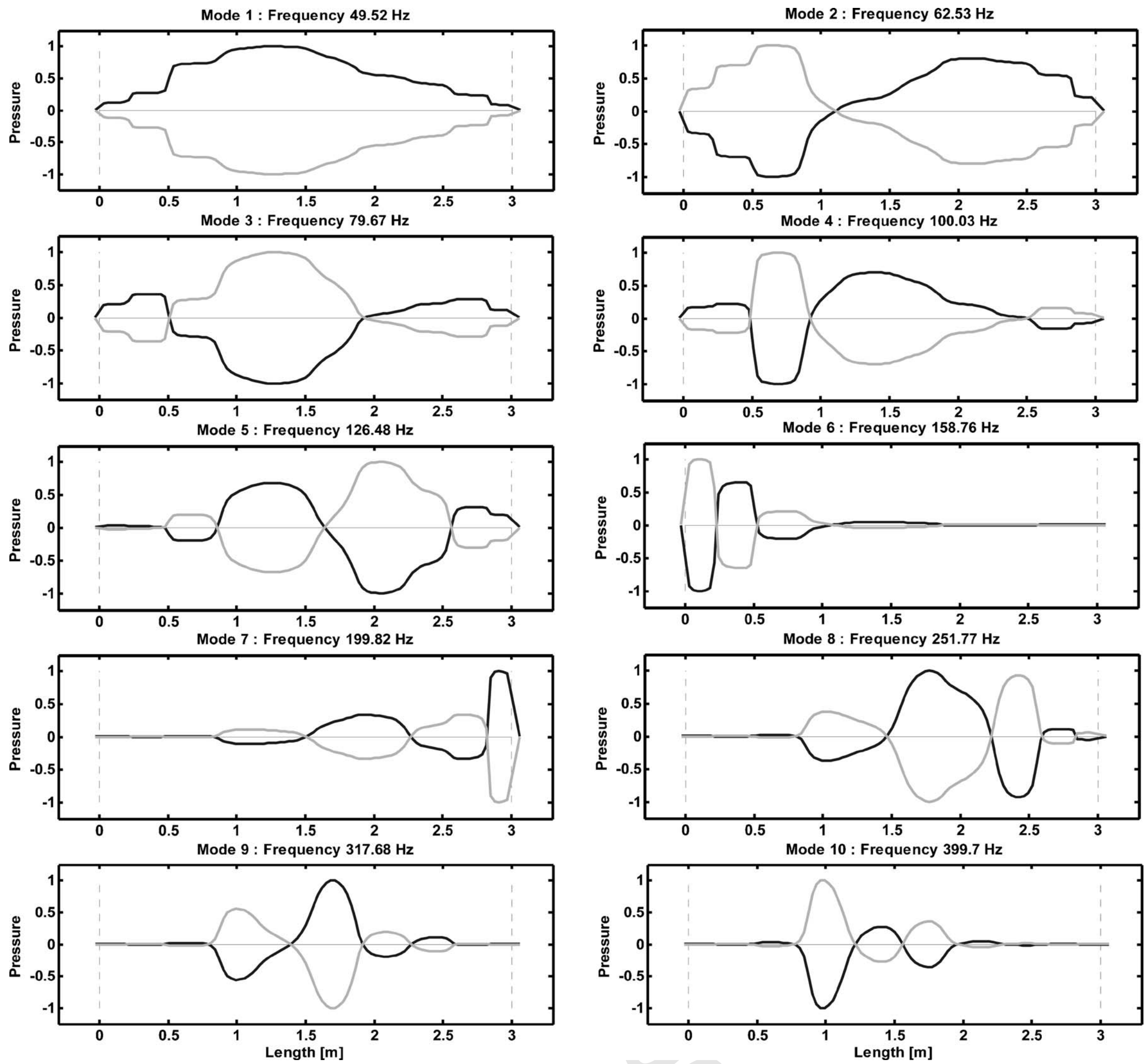

Fig. 10—First 10 pressure mode shapes or resonator (b) in Figure 8.

a target set of modal frequencies characteristic of resonances occurring in control rooms. A computational strategy based on finite element modal calculations coupled with a classical gradient-based optimization approach proved very effective. In particular, smooth shapes and very fast optimizations were

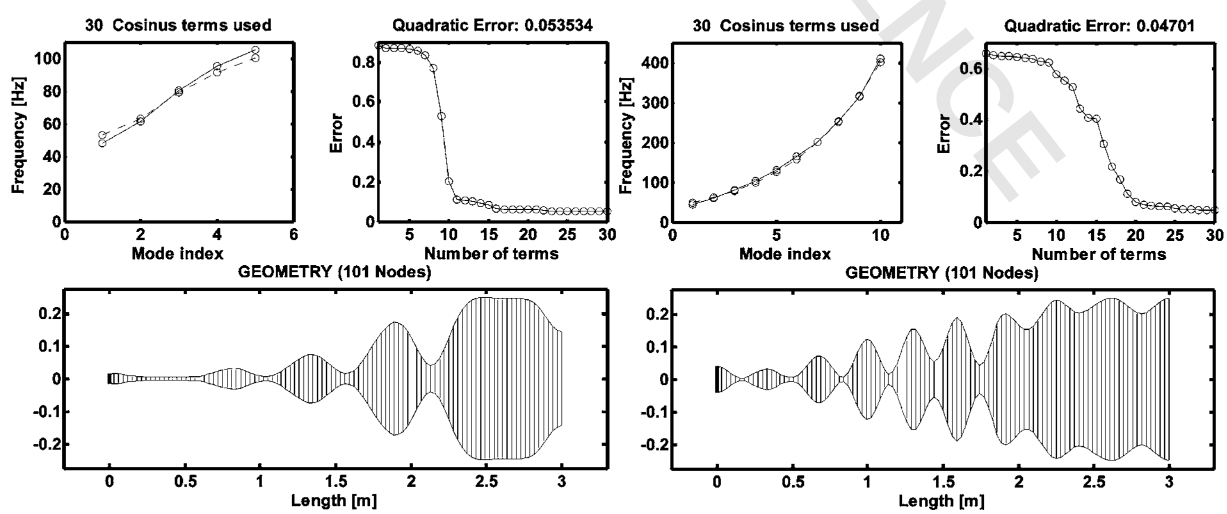

Fig. 11—Resonators optimized to (a) Set 1 and (b) Set 2 using Cosine functions.

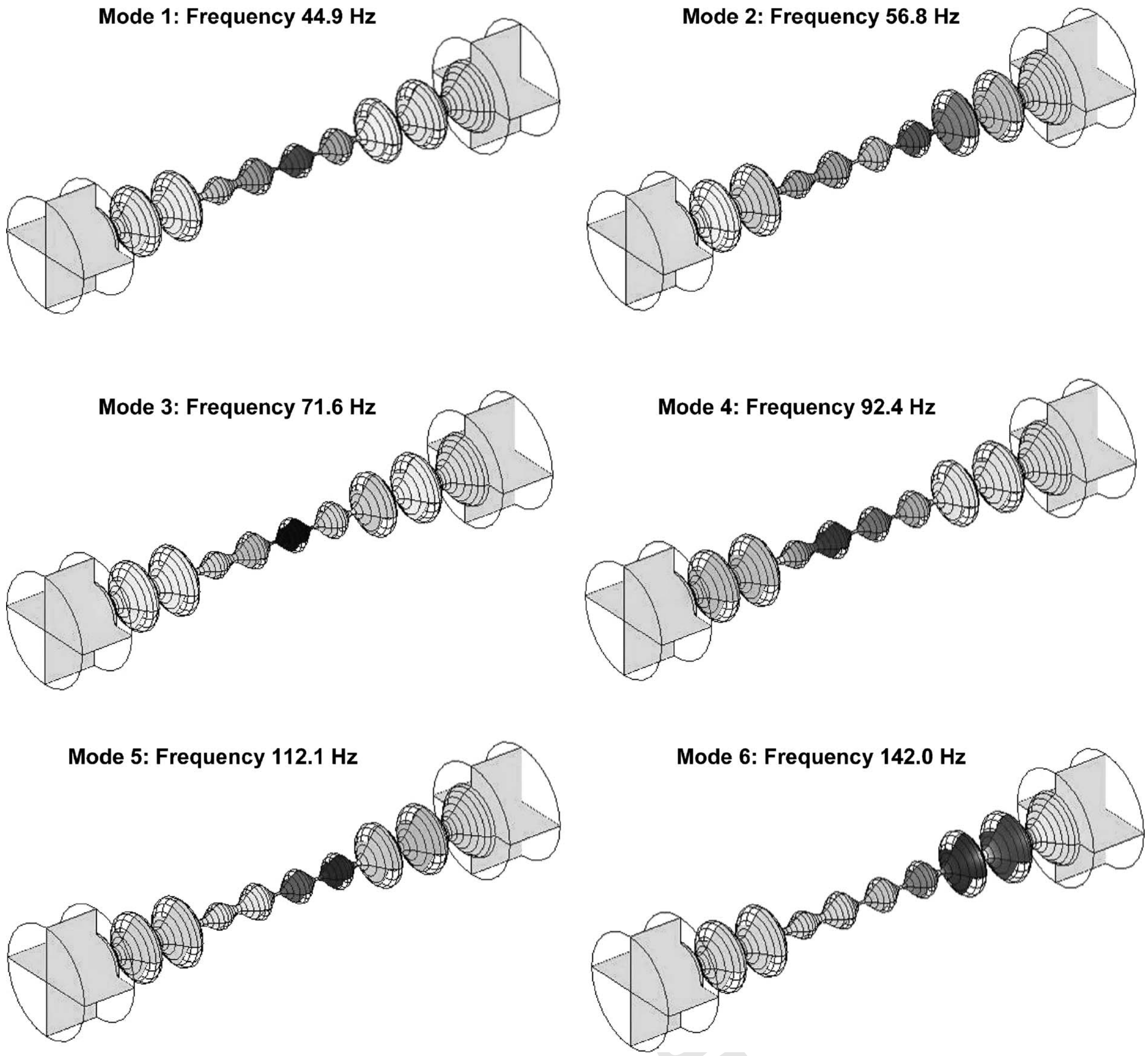

Fig. 12—Three-dimensional finite element modal computations of the resonator shown in Figure 8(a): The normalized modal pressure fields range from maximum depression (blue) to maximum compression (red).

achieved by using various sets of orthogonal functions for describing the geometry.

In this paper, we used a fast 1D finite element acoustical modelling for the optimization procedure. However, additional computations applying a refined 3D model on a couple of optimized resonators showed that, for geometries such as those employed in this paper, the simple 1D model over-estimates modal frequencies by 5 to 20% in the frequency range of interest. Therefore, the results presented in this paper serve to illustrate the proposed optimization methodology, as well as typical resonator shapes which will be obtained. For design purposes, replacing the 1D eigen-computations by a more refined model entrains no further conceptual difficulties, but only a significant increase in the computational load–which can however be accommodated by current technology.

Two different approaches have been suggested to tackle with the problem of undesirable low-frequency resonances: (1) exact resonator mode-matching and (2) evenly spaced resonator modes. Optimized designs have been produced for two different control rooms following both strategies. The numerical results are promising and will be followed by further theoretical analysis of coupled room/resonator systems, as well as experimental work to be reported elsewhere.

8 ACKNOWLEDGMENTS

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9 References

1. H. Helmholtz, On the Sensation of Tone (Dover, New York (1954).
2. U. Ingard, "On the Theory and Design of Acoustic Resonators," *J. Acoust. Soc. Am.* **25**(6), 1037–1061 (1953).
3. R. C. Chanaud, "Effects of Geometry on the Resonance Frequencyof Helmholtz Resonators," *J. Sound Vib.* **178**(3), 337–348 (1994).
4. D. Li and J. S. Vipperman, "On the design of long T-shaped acoustic resonators," *J. Acoust. Soc. Am.* **116**(5), 2785–2792 (2004).
5. F. J. Fahy and C. Schofield, "A Note on the Interaction Between a Helmholtz Resonator and an Acoustic Mode of an Enclosure," *J. Sound Vib.* **72**(3), 365–378 (1980).
6. A. Cummings, "The Effects of a Resonator Array on the Sound Fieldin a Cavity," *J. Sound Vib.* **154**(1), 25–44 (1992).
7. L. Henrique, J. Antunes, "Optimal Design and Physical Modelling of Mallet Percussion Instruments," *Acust. Acta Acust.* **89**, 948–963 (2003).
8. L. Henrique, J. Antunes, O. Inácio, J. Paulino, "Application of optimization techniques for acoustical resonators," *Proceedings 12th ICSV*, , Lisbon (2005).
9. T. Cox and P. D'Antonio, *Acoustic Absorbers and Diffusers–Theory Design and Application*, Spon Press, London, (2004).
10. A. Doria, "Control of Acoustic Vibrations of an Enclosure by means of Multiple Resonators," *J. Sound Vib.* **181**(4), 673–685 (1995).
11. L. Henrique, J. Antunes, J. Carvalho, "Design of Musical Instruments Using Global Optimization Techniques," *Proceedings ISMA 2001*, Perugia (2001).
12. P. Venkataraman, *Applied Optimization with MATLAB® Programming*, Wiley-Interscience, New York (2002).