Iterative method for the remote identification of impact forces at multiple clearance supports using few vibratory measurements

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Abstract Among the wear-prone components in industrial facilities, multi-supported tubes with clearances are particularly critical, as flow excitation may lead to vibro-impact wear between the tubes and their supports. Following our previous studies on remote impact identification using wave-propagation and modal techniques, the approach introduced in this paper consists on an iterative constrained-inversion procedure, using a modal representation of the system, to deal with simultaneous multiple identifications of impact forces, from a limited number of response measurement transducers. Preliminary identification results, based on numerical simulations, assert the satisfactory behaviour of the method to isolate the impact forces in multi-supported systems for realistic noise levels.

Key words: Vibration, impact identification, constrained inversion, regularization

1 Introduction

Identification techniques that enable the diagnostic of real-life industrial components, based on remote vibratory measurements, are quite valuable for validating predictive computations as well as for the control monitoring under real operating conditions. Among the wear-prone components in industrial facilities, gap-supported tubes are particularly critical as flow excitation may lead to vibro-impact wear between the tubes and their supports.Considerable efforts have been invested to enable the identification of impact forces at gap-supports using information from motion transducers located far from the impact locations. Pioneer works theoretical

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and experimental studies by Whiston [1], Jordan and Whiston [2], and Doyle [3]. They presented identifications for simple isolated impacts, achieved in the frequency domain, by using a wave-propagation approach. In a serie of papers [4, 5, 6], the present authors further extended such approach to deal with more complex vibro-impact motions, to move a step closer to realistic conditions. In particular, they achieved, using a pair of vibratory transducers, high quality force identifications for multi-supported tubes, as attested by the validation through laboratory experiments on a beam with three gap-supports.

Following our previous studies on remote impact identification using wave-propagation techniques, we recently applied modal methods to address similar problems [7, 8, 9] which provided quite satisfying identifications on both simulated and experimental data.

The approach introduced here is inspired by identification techniques previously developed by the authors. It consists on an iterative constrained-inversion procedure, based on a modal approach, in order to deal with simultaneous multiple-identifications of impact forces, from a limited number of response transducers. The identifications presented here are performed on numerical simulations of a multi-supported beam with clearances excited by a pulse force. The technique is tested numerically by comparing the identified dynamical impact forces with the actual values stemming from the original nonlinear computations. Preliminary identification results assert the satisfactory behaviour of the method to isolate the impact forces in multi-supported systems for realistic noise levels.

2 The identification problem and procedure

The system addressed consists of a multi-supported beam with clearance supports which displays vibro-impact forces when subjected to a force field. As is typical in real field conditions, we assume a limited set of vibratory transducers to recover the details of the impact forces - less than the number of forces - which makes the problem ill-defined. However, using additional physically-based constraints, we will show that successfull identifications of impact forces can be achieved.

2.1 Modal approach for the force identification

Based on a linear formulation in the frequency domain, the vibratory response $Y_m(\omega)$ of a beam measured at location x_m subjected to an excitation $F_s(\omega)$ located at x_s can be expressed simply as:

$$Y_m(\omega) = H(x_m, x_s, \omega)\, F_s(\omega) \tag{1}$$

where $H(x_m, x_s, \omega)$ is referred to as a transfer function. Adopting a modal representation of the beam and dealing with displacement signals, the *force-to-displacement* function transfer can be built by modal superposition:

$$H^{(d)}(x_m, x_s, \omega) = H^{(d)}_{ms}(\omega) = \sum_{n=1}^{N} \frac{\varphi_n(_m)\varphi_n(x_s)}{m_n(\omega_n^2 - \omega^2 + 2i\omega\omega_n\zeta_n)} \tag{2}$$

where m_n, ω_n, ζ_n and φ_n are the modal parameters, i.e. the modal mass, eigenfrequencies, damping values and mode shapes of the unconstrained beam. The velocity and acceleration responses can be computed similarly to the displacement, using Eq.(1), by replacing $H^{(d)}_{ms}(\omega)$ by the *force-to-velocity* transfer function and the *force-to-acceleration* transfer function given respectively by:

$$H^{(v)}_{ms}(\omega) = i\omega H^{(d)}_{ms}(\omega) \tag{3}$$

$$H^{(a)}_{ms}(\omega) = -\omega^2 H^{(d)}_{ms}(\omega) \tag{4}$$

In the case of a multi-supported beam, several impacts are usually generated at the various support locations. Consequently, the vibratory response measured at location x_m encapsulates the contributions of all the supports. Considering a beam supported at S clearance supports and a set of M of vibratory transducers, Eq.(1) now reads in a matrix form as:

$$\begin{Bmatrix} Y_1(\omega) \\ Y_2(\omega) \\ \vdots \\ Y_M(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & \dots & H_{1S}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) & \dots & H_{2S}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ H_{M1}(\omega) & H_{M2}(\omega) & \dots & H_{MS}(\omega) \end{bmatrix} \begin{Bmatrix} F_1(\omega) \\ F_2(\omega) \\ \vdots \\ F_S(\omega) \end{Bmatrix} \tag{5}$$

While Eq.(5) refers to the direct dynamical problem formulated in the frequency domain, one notices that the inverse problem of finding the excitations $\{F_s(\omega)\}$ from a set of vibratory measurements $\{Y_m(\omega)\}$ is essentially a problem of response inversion. The process of computing the inverse solution appears quite straightforward, except from the fact that, when M<S, a straight inversion is obviously not possible. For the case M=S, the identification procedure starts by computing the frequency-domain vibratory responses by fast Fourier transforming all the measured time-domain signals. Then, one obtains the force estimates - in the frequency domain - from the product of the vibratory responses with the inverse of the transfer operator. Finally, the time-domain identified forces are obtained by inverse Fourier transform. Furthermore, in practice, an important issue is the proper inversion of the transfer operator which describes the phenomena. The ill-conditionning - physical or numerical - of the operator makes inverse problems extremely unstable in that small perturbations can lead to erroneous results and regularization methods are frequently essential to produce usable solutions [10]. Provided the inverse problem is determinate, the previously described basic procedure can be applied to more than one excitation and more than one response measurements using Eq.(5). However, because the number of forces to identifiy is usually higher than the number of response measurements (M<S), the inverse problem is ill-formulated and therefore should be solved differently.

2.2 The proposed iterative multiple-force identification procedure

To deal with the multiple identification inverse problem when M<S and overcome its ill-condition nature, an iterative constrained procedure is now proposed. It operates in an alternate fashion between the time and frequency domains, and enforces some additional physical knowledge for the solution. To be specific, the identification procedure is applied in the frequency domain while physical constraints are imposed to the force estimates in the time domain, at each iteration.

2.2.1 Iterative procedure

Eq.(5) is used as a starting point. Using the information provided by a given vibratory transducers $Y_m(\omega)$, the inverse problem may be formulated in the form:

$$\begin{aligned}
F_1(\omega) &= \frac{1}{H_{m1}(\omega)} Y_m(\omega) - \sum_{s\neq 1}^{S} \frac{H_{ms}(\omega)}{H_{m1}(\omega)} F_s(\omega) \\
F_2(\omega) &= \frac{1}{H_{m2}(\omega)} Y_m(\omega) - \sum_{s\neq 2}^{S} \frac{H_{ms}(\omega)}{H_{m2}(\omega)} F_s(\omega) \\
\vdots & \qquad\qquad\qquad \vdots \\
F_S(\omega) &= \frac{1}{H_{mS}(\omega)} Y_m(\omega) - \sum_{s\neq S}^{S} \frac{H_{ms}(\omega)}{H_{mS}(\omega)} F_s(\omega)
\end{aligned} \tag{6}$$

which is valid for any measurement m and seems well-suited to generate a sequence of improving approximate solutions. One can compute successive approximations of the forces (in the left hand side) from the values obtained in the previous iteration (in the right hand side). By rewritting Eq.(6) in a matrix form and noting the forces to identify $\{F_m(\omega)\} = \{F_1(\omega), \ldots, F_S(\omega)\}^T$, the iterative procedure is governed by:

$$\{F_m(\omega)\}^{i+1} = \{C(\omega)\}Y_m(\omega) - [D_m(\omega)]\{F_m(\omega)\}^i \,, \quad \forall m = 1, \ldots, M \tag{7}$$

where

$$\{C(\omega)\} = \begin{Bmatrix} \frac{1}{H_{m1}(\omega)} \\ \frac{1}{H_{m2}(\omega)} \\ \vdots \\ \frac{1}{H_{mS}(\omega)} \end{Bmatrix}, \text{ and } \quad [D_m(\omega)] = \begin{bmatrix} 0 & \frac{H_{m2}(\omega)}{H_{m1}(\omega)} & \cdots & \frac{H_{mS}(\omega)}{H_{m1}(\omega)} \\ \frac{H_{m1}(\omega)}{H_{m2}(\omega)} & 0 & \cdots & \frac{H_{mS}(\omega)}{H_{m2}(\omega)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{H_{m1}(\omega)}{H_{mS}(\omega)} & \frac{H_{m2}(\omega)}{H_{mS}(\omega)} & \cdots & 0 \end{bmatrix} \tag{8}$$

If two or more response measurements are available, several estimates of the forces can be computed and then compared to isolate the impacts. Actually, when the number of vibratory sensors is equal or greater than the number of forces to identify, successful identifications can be achieved. However, as already mentioned, the problem is not so immediate when M<S.

2.2.2 Identification from the comparison of several measurements

The basic ideas used here, and already discussed in [5] using a different representation formulation for the system dynamics, rely on the following observations: (a) the availability of several estimates of the impact forces; (b) the dispersive nature of the flexural waves. Considering a pair of vibration transducers and a support at a specific location, two estimates of the impact force - stemming from the two measurements - will obviously differ more or less due to the dispersion effect. Since the two inversions are performed with respect to the same support, these estimates should correlate well when impacts arise there and very badly when they were generated elsewhere. It thus suggests that comparing estimates from various measurements is a convenient criterion to isolate impacts generated at distinct clearance supports.
In practice, a narrow moving-window is applied to the two estimates of the force $f_s^{(1)}(t)$ and $f_s^{(2)}(t)$ and then the cross-correlation $\gamma(t)$ is computed. A better estimate $f_s^{\star}(t)$ of the corresponding impact force is then given by:

$$f_s^{\star}(t) = \begin{cases} \dfrac{f_s^{(1)}(t) + f_s^{(2)}(t)}{2} & \text{if } \ \gamma(t) \geq \gamma_c \\ \\ 0 & \text{if } \ \gamma(t) < \gamma_c \end{cases} \qquad \forall s = 1, \ldots, S \tag{9}$$

where γ_c is a parameter which acts as a lower boundary beyond which the constraint given by Eq.(9) is imposed. Obviously, the choice o the moving-window size and γ_c is open to discussion. Here, a value of 5 ms for the window size - which corresponds to the time scale of individual force spikes - works well in practice. As illustrated in [5, 6], the choice of γ_c can be completely automated, but here a fixed value of 0.7 proved to be well suited. Based on the preceeding arguments, the following iterative identification method is then proposed:

1. *Initial identification*

   a. Convert the M available response measurements $Y_m(t)$ to the frequency domain by Fourier transform,
   b. For each support s, compute initial estimates of the impact forces from the M vibratory measurements, each force at a time while ignoring the others, as:

   $$\{F_{s_m}(\omega)\} = \{C(\omega)\} Y_m(\omega)$$

   and convert them to the time domain by inverse Fourier transform,

2. *Identification loop*

   a. Improve the estimates of the impact forces $\{f_{s_m}(t)\}$ by applying the separation constraints according to Eq.(9),
   b. Compute new estimates of the force using Eq.(7) and convert them to the time domain by inverse Fourier transform,
   c. Loop iteratively between tasks 2-a and 2-b,
   d. Assert the convergence of successive iterations by comparing each iteration result with the preceeding identification time domain signals.

The preceeding algorithm seems logical, but an important question concerns the algorithm possible convergence (or not) to a solution. Indeed, problems of noise amplification can lead to instability of the process. However, adequate filtering - as namely SVD filtering or Tikhonov regularization - can overcome such difficulties and stabilize the solutions. Here, a Tikhonov-type regularization is applied to the transfer functions with a regularization parameter of 0.3.

3 Preliminary identification results

To provide a feel for the actual performance of the proposed approach, preliminary identification results are now presented. They are performed on numerical simulations of gap-supported tubes subjected to a pulse force.
Time-domain simulations of gap-supported tubes excited by a pulse were performed using the computational approach described in [8, 11] which has already proved to be adequate to obtain realistic vibro-impact regimes in an effective manner. The modeled beam has length L=6 m, with pinned boundary conditions at both extremities. Two point-supports with symmetrical gap of ±0.5 mm are considered at x_1=1.56 m and x_2=2.64 m. The excitation pulse, originated at t=0.3s, is treated as a third unknown impact force generated at x_3=2 m. The time-domain computations were performed using a modal basis of 9 flexural modes. Their lowest and highest frequencies are 10 and 810 Hz respectively. A constant modal damping of 0.5% was used for all modes. Here, because this paper focus on presenting the technique, the identifications are based on the true modal parameters, the study of the algorithm robustness being postponed to a future paper. The important topic of dealing with the imperfect knowledge of the modal parameters used to build the transfer functions was addressed in [12] where we showed that satisfactory estimates of the modal paramaters can be effectively obtained by optimization techniques.
Figure 1 shows the initial and final identified forces at the three locations, superimposed with the true results. One notices that the initial identifications, obtained after imposing the constraints, are already not so far from the true results even if additional uncorrect features are present. The final identications are, however, undoubtedly cleaner, illustrating the improvement of the identifications by the enforced constraints at each iteration. As can be seen, results compare very well with the computed signals and assert the satisfactory behaviour of the iterative process. Figures 2, 3 and 4 pertain to identifications obtained from noisy measurements. Figure 2 shows the numerical simulated beam reponses, used as inputs for the identification procedure, “measured” at locations 1.07 m and 5.04 m, with noise contamination of about 20% of the RMS magnitude of the corresponding acceleration signals. Figure 3 presents the initial estimates at the three supports computed from each vibratory measurement, before imposing the constraints. For each support, one can notice the influence of the various impacts generated at other supports. From Figure 4, which displays the results after convergence, it is clear that comparing the identifications obtained from the various measurements enables

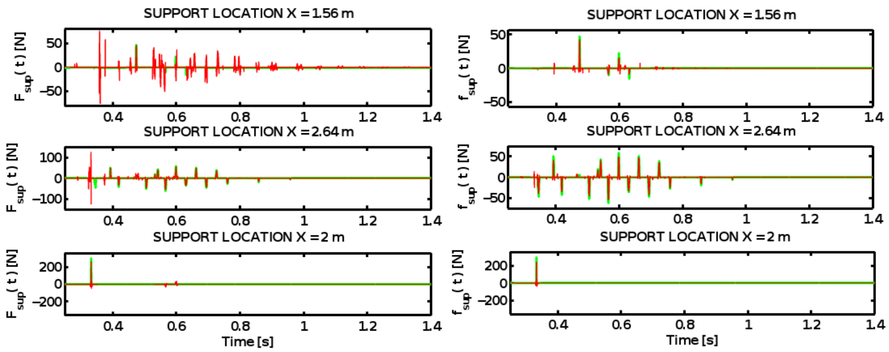

Fig. 1: Simulated (green) and identified (red) impact forces at the three supports obtained by the iterative technique. Left: initial estimates. Right: final identifications.

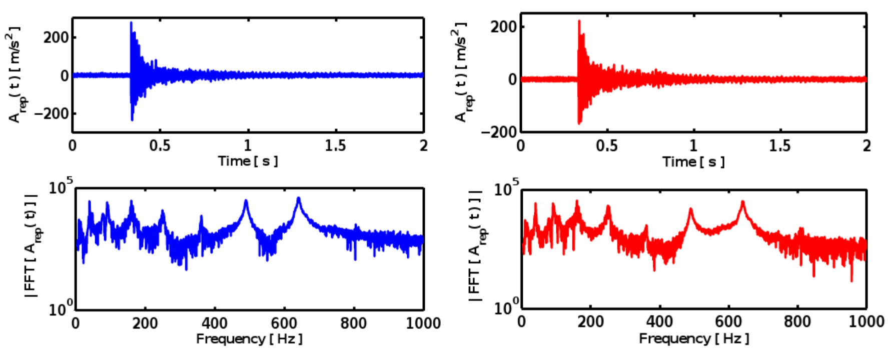

Fig. 2: Time-domain and spectra of the beam acceleration from the two response transducers with noise contamination of 20%. Left: at x =1.07 m. Right: at x =5.04 m.

a good discrimination of the impacts originated at different locations. Finally, in Figure 4, even if the true and identified forces are overall similar, significant details - including impacts - are missing in the identifications as the result of noise. Moreover, some residuals, which have not been effectively eliminated by the constraints procedure, are still present. There is therefore room for improvement of the proposed technique.

4 Conclusions

In this paper, we proposed an iterative-constrained inversion technique from a limited number of response transducers, based on a modal approach, to address the problem of simultaneous multiple impacts in a multi-supported beam. Illustrative identifications show that impact force can be attempted with success for a beam excited by a pulse force for realistic noise levels. Future work will address more

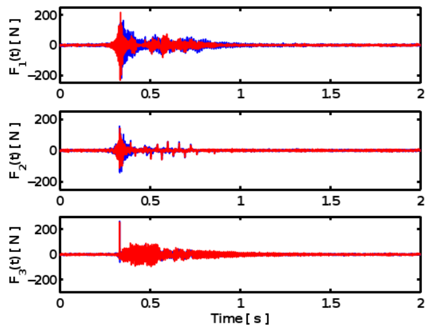

Fig. 3: Initial estimates of the impact forces at the three supports from measurements at x = 1.07 m (blue) and x = 5.04 m (red).

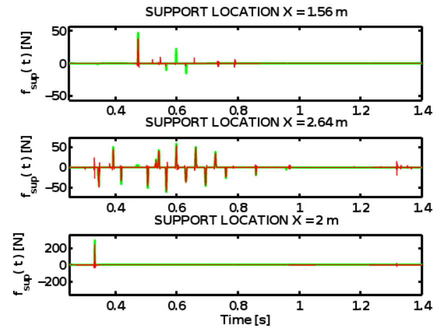

Fig. 4: Simulated (green) and identified (red) impact forces obtained after convergence, with noise contaminated measurement of 20%.

difficult problems of practical interest, such as a multi-supported beam subjected to turbulence flow excitation as is typical of heat-exchangers.

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