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APPLICATION OF OPTIMIZATION TECHNIQUES FOR ACOUSTICAL RESONATORS

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Abstract

Following our preceding work on the optimization of structural vibrating components of musical instruments, we now have extended those techniques to acoustical resonators for marimba-type instruments or other applications. In this paper we suggest that acoustical resonators may be tuned to multiple modes of the vibrating bars, in order to enrich or simply modify the timbre of the instrument.

After a short review of our modelling and optimization techniques, we illustrate the approach with several examples. A detailed study of the acoustical properties of three optimized resonators with various relationships of the optimized acoustic modal frequencies is presented, based on finite-element acoustic computations. The shapes of such multi-modal optimized resonators are completely different from the typical cylindrical resonators. Finally, we present experimental results which validate the theoretical approach.

INTRODUCTION

In a previous paper Henrique *et al.* [1] developed a method for the optimal design of percussion instrument bars, such as found in xylophones, marimbas and vibraphones. The aim was to compute optimal bar shapes, in order to comply with a pre-defined target set of modal frequencies, shaping the spectral content of the instrument response, for a given number of geometrical constraints.

Traditionally, these percussion musical instruments have individual acoustical resonators coupled to each bar in order to reinforce the sound level. This effect is obtained by resonance of the air cavity inside a (usually cylindrical) tube whose first acoustical modal frequency is chosen to coincide with the first vibratory modal frequency of the respective bar.

Typically, the bar modal frequencies of interest will display harmonic relationships. For instance, usual values of the first and second modal frequencies for xylophones and marimbas attempt integer relationships of 1:3 and 1:4, respectively, with some variations on the third flexural modal frequency, most usually between 1:9 and 1:10. It is well known that common closed-open instrument resonators will only reinforce modes with odd frequency relationships. Therefore, in order to enhance the effectiveness of the resonance phenomenon and enrich or simply modify the timbre of the instrument, it may prove extremely interesting to devise new resonator shapes which are capable to reinforce all the tuned modes in a given instrument.

In this paper we extend the techniques developed in [1-3] to acoustical resonators of marimba-type instruments, however other applications may also be addressed. After a short review of our modelling and optimization techniques, we illustrate the approach with three resonators with various relationships of the optimized acoustic modal frequencies. The shapes of such multi-modal optimized resonators are completely different from the typical cylindrical resonators. A detailed study of the acoustical properties of the optimized resonators is presented, based on finite-element acoustical computations. Finally, we present experimental results which validate the theoretical approach.

ACOUSTICAL MODELLING OF THE RESONATORS

The sound propagation model used in this paper is based on the mono-dimensional wave equation, for tubes of variable cross-section $S(x)$ along their axis. The numerical computation of these continuous systems can be obtained by discretization of the geometry in N finite conical elements of shape $S_e(x)$ characterized by a transverse section S_1 at the start of the element and S_2 for the section in the other extremity. For each conical finite element the sound propagation can be described by the Webster equation [4]:

$$\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S_e(x)} \frac{\partial}{\partial x}\left(S_e(x) \frac{\partial p}{\partial x} \right) = 0 \tag{1}$$

The change of pressure inside the element can be described as a linear polynomial of the first order $p(x,t) = a_0 + a_1 x$, where the coordinate x is understood as local (respectively $x = 0$ and $x = L_e$ in the two nodes of each element). We can derive an approximate solution for $p(x,t)$, which satisfies equation (1) in terms of a residual term $R(x,t)$ to be minimized. Using the Galerkin method we obtain:

$$\int_0^L R(x,t) N_n(x)\, dx = 0 \quad \Rightarrow \quad \int_0^{L_e} \{N(x)\} \left[\frac{\partial^2 p}{\partial t^2} - \frac{c^2}{S_e(x)} \frac{\partial}{\partial x}\left(S_e(x) \frac{\partial p}{\partial x} \right) \right] dx = 0 \tag{2}$$

where $N_n(x)$ is the weighting function of the spatial approximation and $\{N(x)\}$ is the corresponding weighting vector derived from the polynomial coefficients. After the necessary integrations, we obtain:

$$\left[M_e\right]\left\{\ddot{P}(t)\right\}+\left[K_e\right]\left\{P(t)\right\}=0 \tag{3}$$

where $\left\{P(t)\right\}$ is the vector describing the pressure at each node of the element. The elementary matrices of mass and rigidity are obtained as

$$\left[M_e\right]=\frac{\rho S_1 L_e}{12}\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}+\frac{\rho S_2 L_e}{12}\begin{bmatrix}1 & 1\\ 1 & 3\end{bmatrix};\quad \left[K_e\right]=\frac{\rho c^2\left(S_1+S_2\right)}{2L_e}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \tag{4,5}$$

For the global system, these elementary matrices are assembled as usual. The procedure described in this section allows the computation of the modal frequencies which will be used in the optimization iterations to find the desired shape of the resonator.

OPTIMIZATION PROCEDURES

Many parameters are involved in a geometry optimization problem, with two unwanted consequences: Firstly, the optimization becomes computationally intensive, and this is further true as the number of parameters to optimize P_p ($p=1,2,\cdots$) increases. Secondly, the error hyper-surface $\mathcal{E}(P_p)$ where the global minimum is searched will display in general many local minima.

In [2] we avoided converging to sub-optimal local minima by using a robust (but greedy) global optimization technique – simulated annealing [5]. In order to improve the computational efficiency, the global optimization algorithm was coupled with a deterministic local optimization technique [5], to accelerate the final stage of the convergence procedure. Very encouraging results have been obtained, demonstrating the feasibility and robustness of this approach, as well as the potential to address other aspects of musical instrument design. However, a negative side effect was the need for significant computation times, which seem ill-suited to the optimization of large-scale systems – such as, for instance, carillon bells.

More recently, we alleviated this problem by significantly reducing the dimension of the search space where optimization is performed [1,3]. This can be achieved in several ways, by describing the geometrical profiles of the vibrating components in terms of a limited number of parameters. Here, we chose to develop $S(x)$ in terms of a set of orthogonal characteristic functions $\Psi_s(x)$, such as Tchebyshev polynomials or trigonometric functions, optimizing their amplitude coefficients. For complex systems, described by finite-element meshes with hundreds or thousands of elements, this approach reduces the size of the optimization problem by several orders of magnitude. Then, we have found that, most often, acceptable solutions can be obtained using efficient local optimization algorithms, leading to a further reduction in computation times. The examples presented in this paper have been obtained using such approach, as described in [1].

Error Function

In an optimization problem the objective is generally to find the values of a set of variables describing a system that maximizes or minimizes a chosen error function, usually satisfying a set of imposed restrictions. In the present case, we wish to find the optimal shape of the resonator, described by its variable cross section $S(x)$ and length L which minimizes the deviations from the computed modal frequencies

$\omega_m[S_e(x), L]$ and the reference target set ω_m^{ref}. This error function will be formulated as:

$$\mathcal{E}[S_e(x), L] = \sum_{m=1}^{M} W_m \left\{ 1 - \frac{\omega_m[S_e(x), L]}{\omega_m^{ref}} \right\}^2 \quad (6)$$

where W_m are weighting factors for the relative modal errors and M is the number of modes to optimize.

OPTIMIZATION RESULTS

In this section we present some examples of resonators of circular section, optimized using the technique of geometric description in terms of characteristic functions, coupled with a deterministic optimization algorithm with constraints.

All the resonators showed in this section were optimized to a fundamental frequency of 220 Hz, with different frequency relations of several modes. Figure 1 shows the convergence of the optimization procedure as the number of characteristic functions for the shape description is increased. In this specific example (tuning 1:3:5:8), convergence is obtained after 6 Tchebyshev polynomials are used.

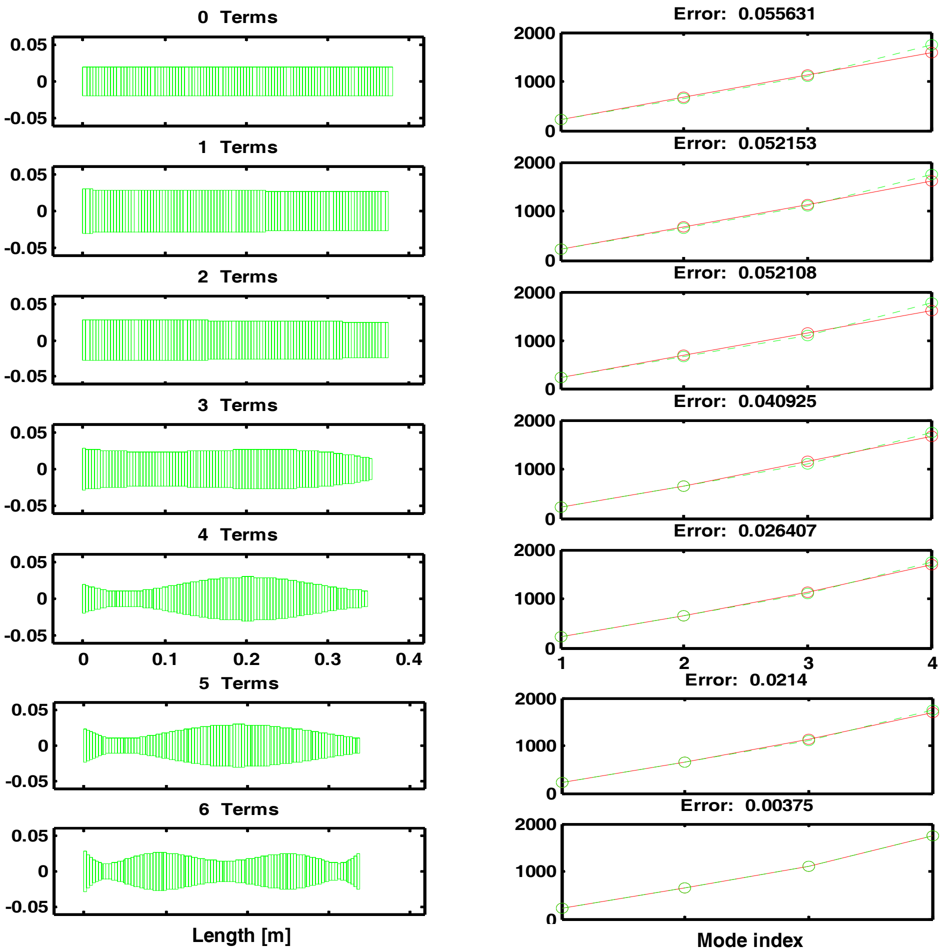

Figure 1 – Optimized resonator for the frequency 220 Hz with the partials 1:3:5:8. Convergence of the optimization process with the increase of the number of characteristic functions used.

The examples of Figures 2(a) and 2(b) show two resonators optimized for relations 1:3:5:8, the first computation using Tchebyshev polynomials as characteristic functions, and cosinusoidal functions for the second example.

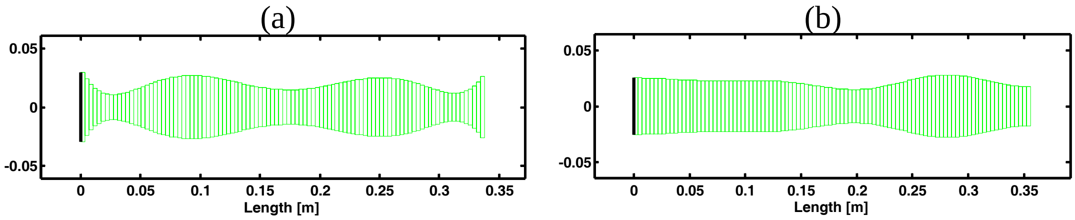

Figure 2 – Optimized resonator for the frequency 220 Hz with the partials 1:3:5:8. Optimization achieved using (a) Tchebyshev polynomials and (b) Cosines as characteristic functions.

Notice that, usually, the resonators obtained using different characteristic functions present different shapes. This observation highlights the fact that the optimization problem does not present a single solution. In other words, several geometries can display identical modal frequencies for the modes that are optimized. However, as it is obvious, modeshapes will be different. And, for the other modes not included in the objective-function to minimize, the resonators will present different frequencies. The computations in Figures 3(a) and 3(b) depict optimized resonators for relations 1:3:8:12 and 1:4:6:10, leading to different bore profiles.

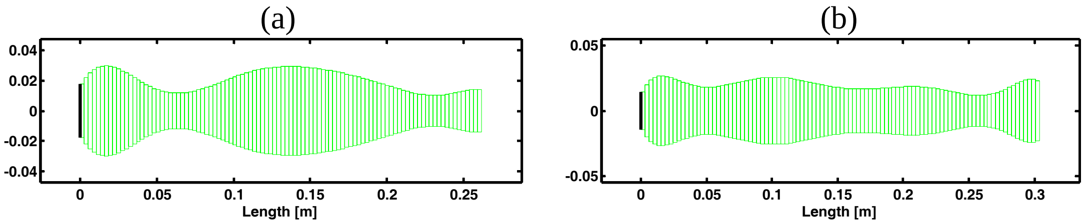

Figure 3 – Optimized resonators for the frequency 220 Hz obtained using Tchebyshev polynomials, with partials (a) 1:3:8:12 and (b) 1:4:6:10.

Figure 4 illustrates the pressure modeshapes of the optimized modes for the geometry shown in Figure 2(a).

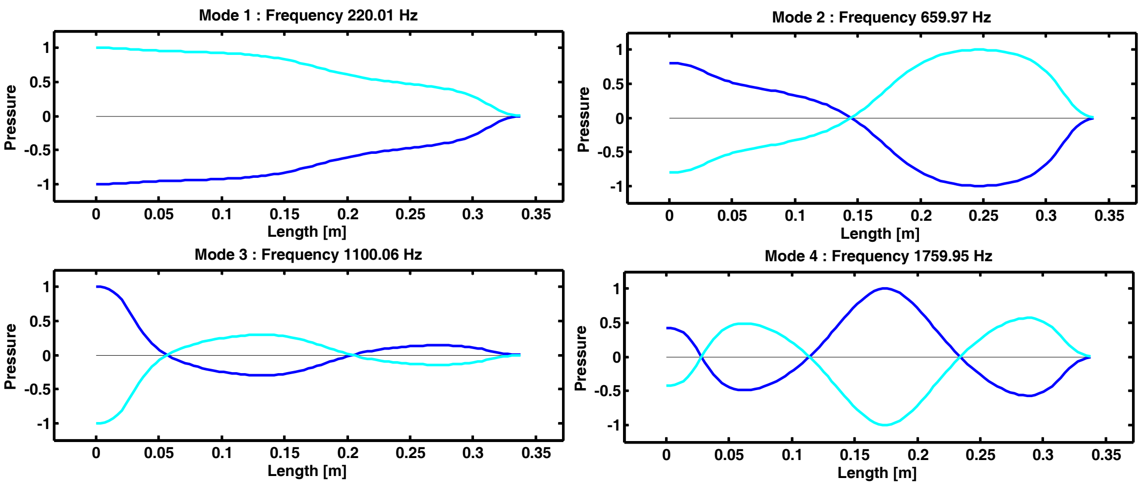

Figure 4 – Pressure modeshapes of the first four modes of the optimized resonator for the frequency 220 Hz with the partial 1:3:5:8 pertaining to Figure 2(a).

EXPERIMENTAL VALIDATION

In this work, the experimental validation was performed by building three closed-open resonator-prototypes of Plexiglas, tuned for different combinations of partials. The partials were chosen to coincide with the tuning of three bars that were previously constructed (1:4:10; 1:1.67:4:10 and 1:4:6.67:10). Besides these optimized resonators, a traditional cylindrical resonator was also built, for preliminary tests of the measuring setup and for comparison with the optimized resonators. The prototype resonators were build by piling up Plexiglas boards with 7.74 mm thickness (the same of the finite elements used in computations), in which conical holes were machined with dimensions stemming from the optimization computations.

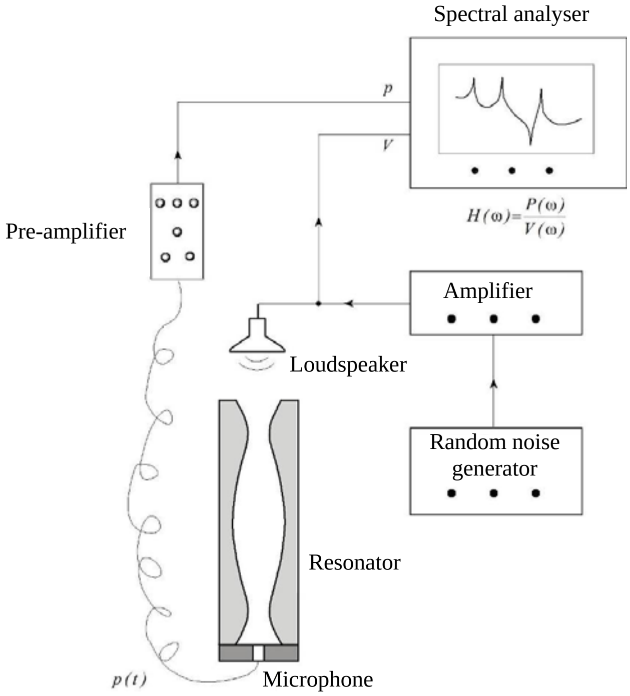

Figure 5 – Experimental setup used for the modal identification of the optimized resonators.

Experimental Setup

For the acoustical tests and identification of the modal properties of the resonators we used the experimental setup shown in Figure 5. The resonators were excited using an external speaker, emitting a white noise signal supplied by a generator and then amplified. The acoustic pressure at the end of the resonators, sensed by the microphone, together with the driving signal of the speaker, allowed us to obtain transfer functions of acceptable quality. The three prototype resonators tested can be observed in Figure 6.

It will be interesting to notice that, for a same fundamental frequency, the optimized resonator 1:4:10 is quite smaller (approximately 180 mm) than the traditional cylindrical resonator (approximately 290 mm). This fact constitutes a significant additional advantage of many optimized resonators.

Figure 6 – Optimized resonators. From left to right: 1:4:10; 1:1.67:4:10 and 1:4:6.67:10.

Discussion of Results

Optimization of the experimental resonators was performed for a frequency tuning of 285 Hz. Table 1 summarizes the values of the computed and identified frequencies, from the measured transfer functions shown in Figure 7. This table reveals that the frequency ratios of the optimized modes were, for all the resonators, extremely satisfying, validating the methodology developed in this paper.

On the other hand, concerning the absolute frequencies, small deviations were observed in resonators 1:4:10 and 1:1.67:4:10, which sound about 3% below the target frequencies. These measurements were performed at temperature 20ºC, the same postulated in the computations ($c = 343$ m/s), so that the ascertained differences cannot be justified from the temperature effect. Indeed, three dimensional finite element computations performed on the optimized geometries have showed that the small errors obtained are mostly due to a slight inadequacy of the mono-dimensional wave model when dealing with complex geometries. On the other hand, the geometry of the test models was such that the open end of the resonators behaves somewhat between a flanged and a non-flanged termination.

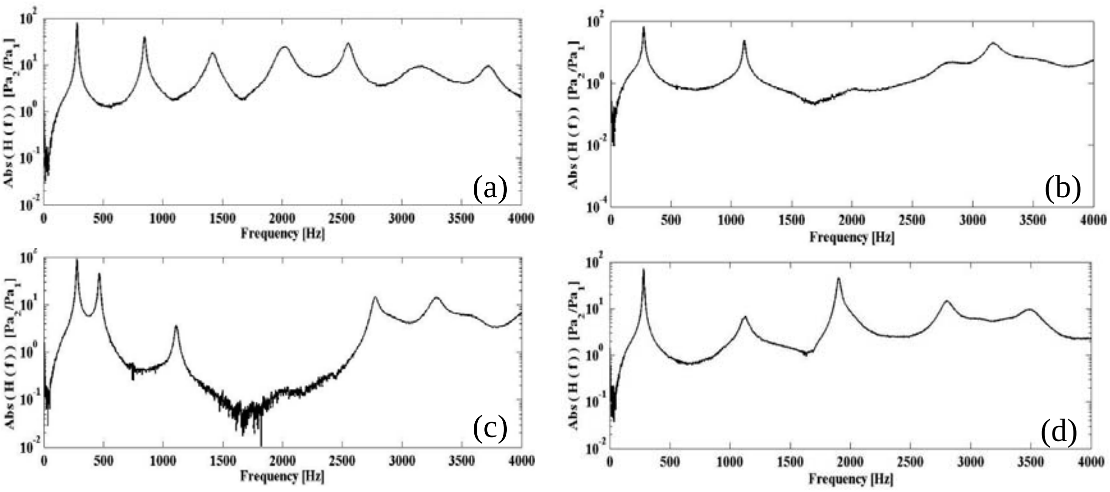

Figure 7 – Measured transfer functions, respectively for the (a) cylindrical resonator (1:3:5:7:…) and for the optimized resonators: (b) 1:4:10; (c) 1:1.67:4:10 and (d) 1:4:6.67:10.

Table 1 – Computed and experimental values of the frequencies of the first modes and corresponding frequency ratios of the optimized open-closed resonators.

Tuning	Mode	Computed Modal Frequencies (Hz)	f_i/f_1	Measured Modal Frequencies (Hz)	f_i/f_1
1:4:10	1	278.01	1.00	275.4	1.00
	2	1103.0	3.97	1109.8	4.03
	3	2579.9	9.28	2773.6	10.07
1:1.67:4:10	1	280.9	1	277.7	1.00
	2	469.0	1.67	465.2	1.68
	3	1115.9	3.97	1109.8	4.00
	4	2705.5	9.63	2775.5	9.99
1:4:6.67:10	1	283.7	1.00	284.3	1.00
	2	1159.8	4.09	1136.6	4.00
	3	1882.1	6.63	1875.6	6.60
	4	2737.7	9.65	2858.2	10.05

CONCLUSIONS

In this paper we presented an effective technique for the shape optimization of resonators in order to obtain a target set of modal frequencies. A computational strategy based on a mono-dimensional wave propagation model coupled with a classical gradient-based optimization approach proved very effective. In particular, smooth shapes and very fast optimizations were achieved by using various sets of orthogonal functions for describing the geometry.

Several illustrative examples have been presented, which show the fast convergence of the results with just a few characteristic functions. Also, it has been shown that different families of characteristic functions may lead to different optimized shapes, as solutions are not unique. Overall, when unorthodox tuning frequency ratios are targeted, the shapes of the optimized resonators are very different from those typically found in musical percussion instruments.

The theoretical results obtained were confronted to experimental measurements, performed for three different resonators, which were built and tested. Accounting for the limitations of the mono-dimensional propagation model and incertitude in the boundary conditions at the open extremities, the results obtained are very satisfactory; the maximum error observed being about 3% with respect to the nominal values. The proposed methodology has been therefore validated. Other applications of the proposed methodology may be suggested as shown in [6].

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